# Pb-Bi EutecticViscosity

## Preliminary Recommendation

The preliminary recommendation for an equation to represent the viscosity of Pb-Bi eutectic from the melting point, 125ºC (398 K) to 1000ºC (1273 K) is:

$$\eta = 0.490 \exp\left[\frac{760.1}{T}\right] \quad \textbf{(1)}$$

where η is the dynamic viscosity in centipoise (mPa · s ) and T is the temperature in K. This equation was obtained by a least square fit of the data of Kaplun et al. [1], the IPPE data reported by reported by Nikol'skii et al. [2], and the data of Bienas and Sauerwald [3]. Figure 1 shows this recommended equation, with uncertainty bands and the available data [1-4].

## Uncertainty

The uncertainty in the values calculated from Eq.(1) are assumed to be ± 6%. This uncertainty is the square root of the sum of the squares of the uncertainty in the measurements and the uncertainty in fitting the data. The measurement uncertainty has been taken as ± 5%, the uncertainty reported by Kaplun et al. [1]. Because no measurement uncertainties were reported by either Nikol'skii et al. [2] or Bienas and Sauerwald [3], the uncertainty reported by Kaplun et al. [1] was assumed for these data. The uncertainty in fitting the data is the average percent deviation of the data from the equation, which is about 3%.

## Discussion

### *Review of Data*

The available data [1-5] on the viscosity of lead-bismuth eutectic are shown in Figure 2 and listed in Table 1. Table 1 lists the year of measurements, temperature range, number of data, and composition in weight percent. All the data reported in the Liquid Metal Handbook [4] are based on the measurements by Bienas and Sauerwald [3] except for the single datum at 332 ºC (605 K),

which is from a report by the American Smelting and Refining Co. [5]. This datum, shown in Figure 2, has not been included in the analysis because no information could be obtained about its origin. It is unclear whether this datum represents and experimental measurement, a calculated value based on lead and bismuth viscosities, or a best estimate. The data reported by Kutateladze [6] are identical to the data given by Nikol'skii et al. [2].

**Table 1 Pb-Bi Viscosity Data**

| Experimenter | Year | Temperature, K | No. of Points | Composition, wt% |
|---|---|---|---|---|
| Kaplun [1] | 1979 | 394-1180 | 98 | 44.5% Pb- 55.5% Bi |
| | | 396-1167 | 177 | |
| IPPE, Nikol'skii [2] | 1955 | 423-1137 | 25 | 44.5% Pb - 44.4% Bi |
| Bienas & Sauerwald [3] | 1927 | 676-1107 | 4 | 51.3% Pb - 48.7% Bi |
| Lyon (Am. Smelting & Refining Co.) [4,5] | 1948 | 605 | 1 | 51.3% Pb - 48.7% Bi |

***Data Analysis***

In analysis of these data, all the temperatures were converted to the 1968 International Practical Temperature Scale (IPTS), which is identical to the 1990 International Temperature Scale for the temperature ranges of these data.

Kaplun et al. [1] fit their 215 data to the equation:

$$\eta = 0.4656 \exp\left[\frac{773.2}{T}\right] \qquad \textbf{(2)}$$

which has a temperature dependence suggested by Andrade [7]. In analysis of their data on bismuth, lead, and zinc, Ofte and Wittenberg [8] concluded that their data on these pure metals are fit better

with equations of the form

$$\ln\left(\frac{\eta}{T}\right) = \left(\frac{A}{T}\right) + B \qquad \textbf{(3)}$$

rather than the Andrade form

$$\ln(\eta) = \left(\frac{A}{T}\right) + B \qquad \textbf{(4)}$$

Their plots of log (η /T) vs 1000/T gave straight lines whereas there was a slight curvature in plots of log (η) vs 1000/T. Both forms of equations have been considered for the viscosity of Pb-Bi eutectic. Graphs of ln (η /T) vs 1000/T and ln (η) vs 1000/T are shown, respectively, in Figures 3 and 4. There is curvature in the plot of ln (η /T) vs 1000/T (Figure 3) whereas the plot of ln (η) vs 1000/T (Figure 4) approximates a straight line, indicating this is the correct temperature dependence. Consequently, the Andrade equation has been used to fit the data on the viscosity of Pb-Bi eutectic.

Table 1 shows that the two sets of measurements by Kaplun et al.[1] have many more points than the other measurements included in this analysis. To prevent biasing the least squares fit by these data sets, a weight has been used in calculating the variance. The square of the deviation of the fit from each datum has been weighted by the inverse of the number of points in the data set to which that datum belongs. Eq.(1) was obtained from a weighted least squares fit to the two data sets of Kaplun et al. [1], the IPPE data reported by reported by Nikol'skii et al. [2], and the data of Bienas and Sauerwald [3]. The data fit and recommended equation are shown in Figure 5. Figure 1 shows the recommended equation, all available data, and the uncertainties. The uncertainties are the sum of the squares of the uncertainty in the fit and the experimental uncertainties. Note that although the

datum given by Lyon has not been included in the fit, it is consistent with the recommended equation. Values obtained with the recommended equation are tabulated as a function of temperature in Table 2. Figure 6 compares the recommended equation with the equations given by Kaplun et al.[1] and the tabulated values of Nikol'skii et al.[2].

**Comparison with ATHENA Viscosity Equations**

The ATHENA code [9] is being used in the thermal-hydraulic design and analysis of Pb-Bi cooled reactors. Lead-bismuth properties have been added to the ATHENA code for these analysis. The ATHENA code calculates the dynamic viscosity ($\eta$) of Pb-Bi eutectic from the kinematic viscosity ($\nu$) and the density ($\rho$) using the relation:

$$\eta = \nu\rho \qquad \textbf{(5)}$$

where

$$\nu = 61.423(T-273.15)^{-0.61106}\cdot 10^{-7} \qquad \textbf{(6)}$$

and

$$\rho = 10728.0 - 1.2159(T-273.15) \qquad \textbf{(7)}$$

In Figure 7, viscosity values calculated with the ATHENA code equations [ Eq.(5-7)] are compared with the recommended equation, Eq.(1), and with the available data. From 125 to 800ºC, viscosities obtained from the ATHENA equation, which is based on the IPPE data [6,2], are high relative to values from the recommended equation. Above 800ºC, the ATHENA equation intersects the recommended equation. Figure 8, which includes the uncertainties in the recommended equation, shows that, for the most part, the ATHENA values are just within the upper uncertainties.

**Table 2 Recommended Values for the Dynamic Viscosity of Pb-Bi Eutectic**

| Temperature K | Temperature °C | Viscosity mPa s |
|---|---|---|
| 400 | 126.85 | 3.28 |
| 450 | 176.85 | 2.65 |
| 500 | 226.85 | 2.24 |
| 550 | 276.85 | 1.95 |
| 600 | 326.85 | 1.74 |
| 650 | 376.85 | 1.58 |
| 700 | 426.85 | 1.45 |
| 750 | 476.85 | 1.35 |
| 800 | 526.85 | 1.27 |
| 850 | 576.85 | 1.20 |
| 900 | 626.85 | 1.14 |
| 950 | 676.85 | 1.09 |
| 1000 | 726.85 | 1.05 |
| 1050 | 776.85 | 1.01 |
| 1100 | 826.85 | 0.98 |
| 1150 | 876.85 | 0.95 |
| 1200 | 926.85 | 0.92 |
| 1250 | 976.85 | 0.90 |
| 1300 | 1026.85 | 0.88 |

$1cP = 1\ mPa\ s = 10^{-2}\ g\ cm^{-1}\ s$

**References**


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4. R. N. Lyon, ed., Liquid-Metals Handbook, pp. 42-43, 56, 99-102, U.S. Atomic Energy Commission (1952, 1954).

5. American Smelting and Refining Co. Reports on N8onr-644, Nos. N1, N3, N4, N5, and N6 (194801950); as referenced by Lyon [ref. 4].

6. S. S. Kutateladze, V. M. Borishanskii, I. I. Novikov, and O. S. Fedunskii, *Liquid-Metal Heat Transfer Media,* Supplement No. 2, of the Soviet Journal of Atomic Energy, Atomic Press, Moscow, 1958 [English translation, 1959].

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**Fig. 1 Pb-Bi Eutectic DynamicViscosity**

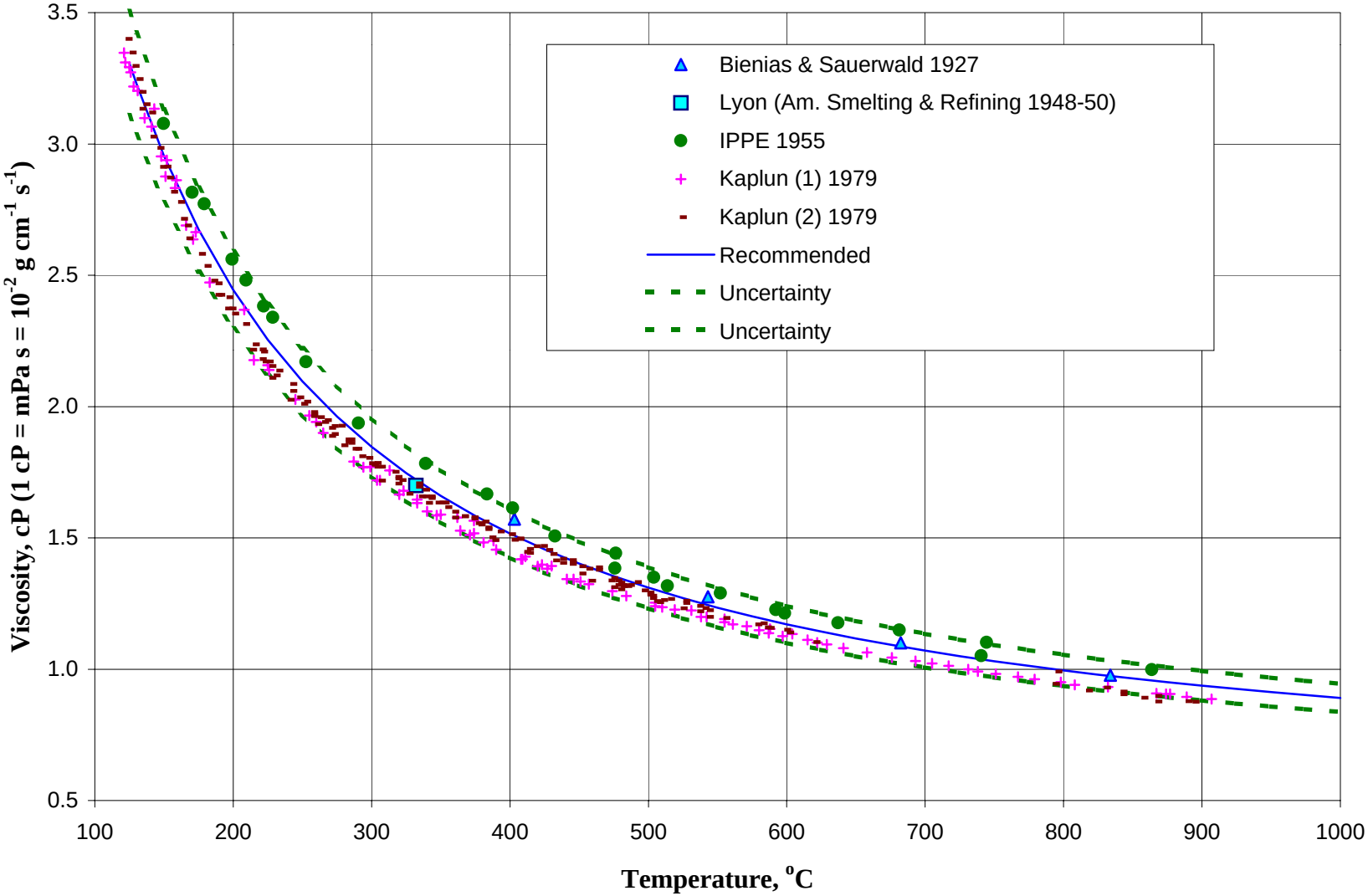

**Fig. 2 Pb-Bi Eutectic DynamicViscosity Data**

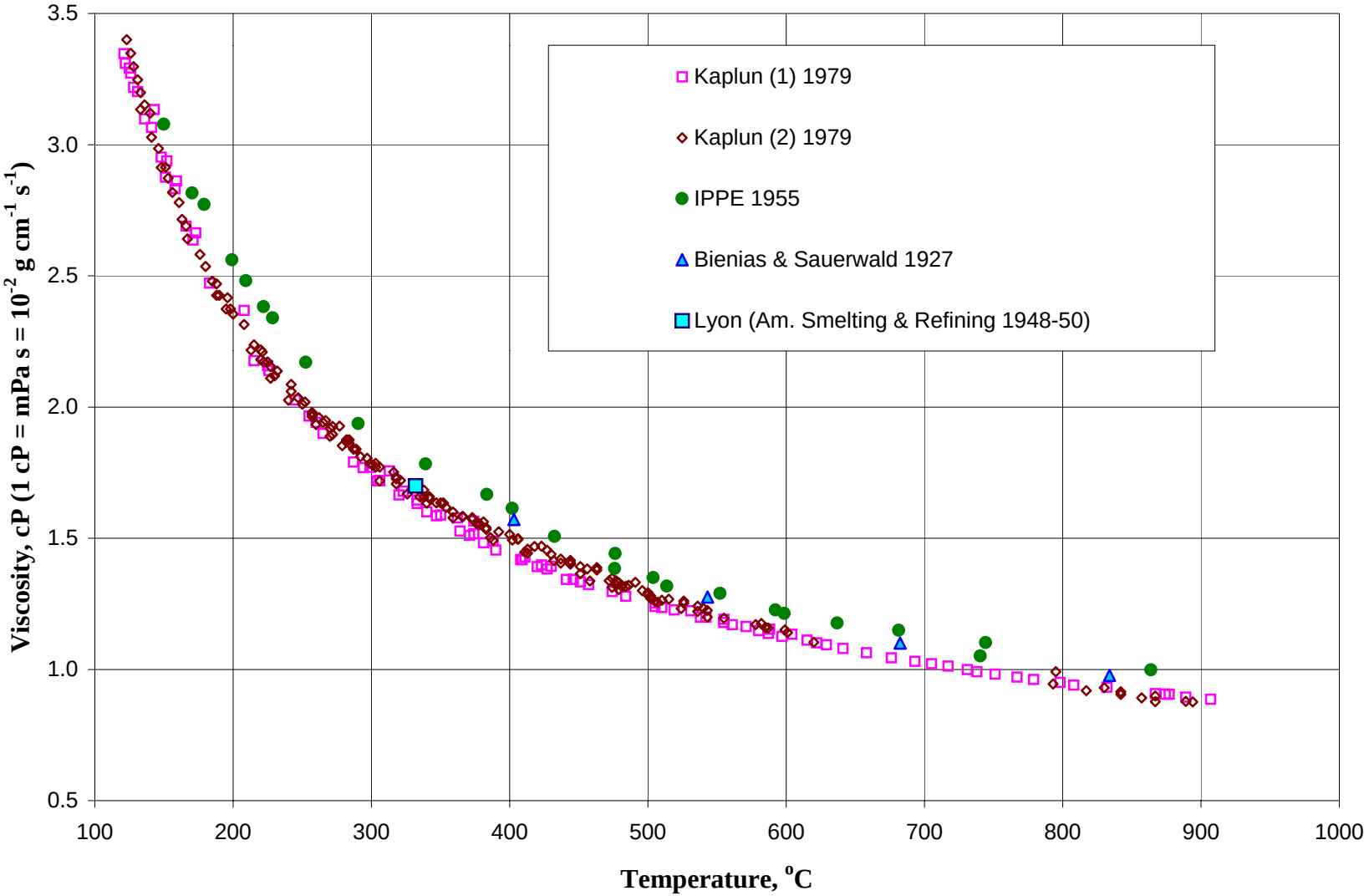

**Fig. 3 Natural logarithm of viscosity/T**

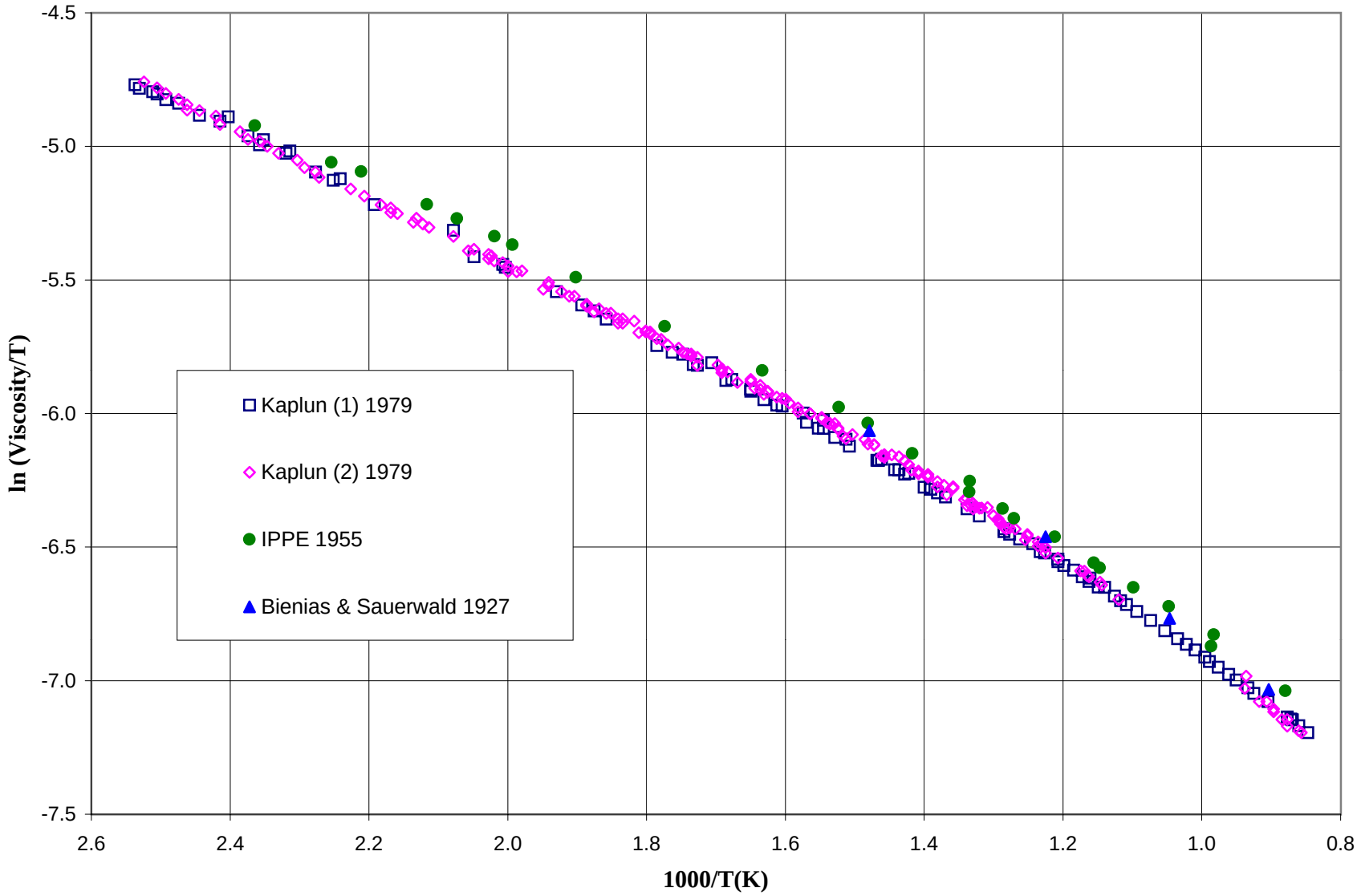

**Fig. 4 Natural logarithm of viscosity**

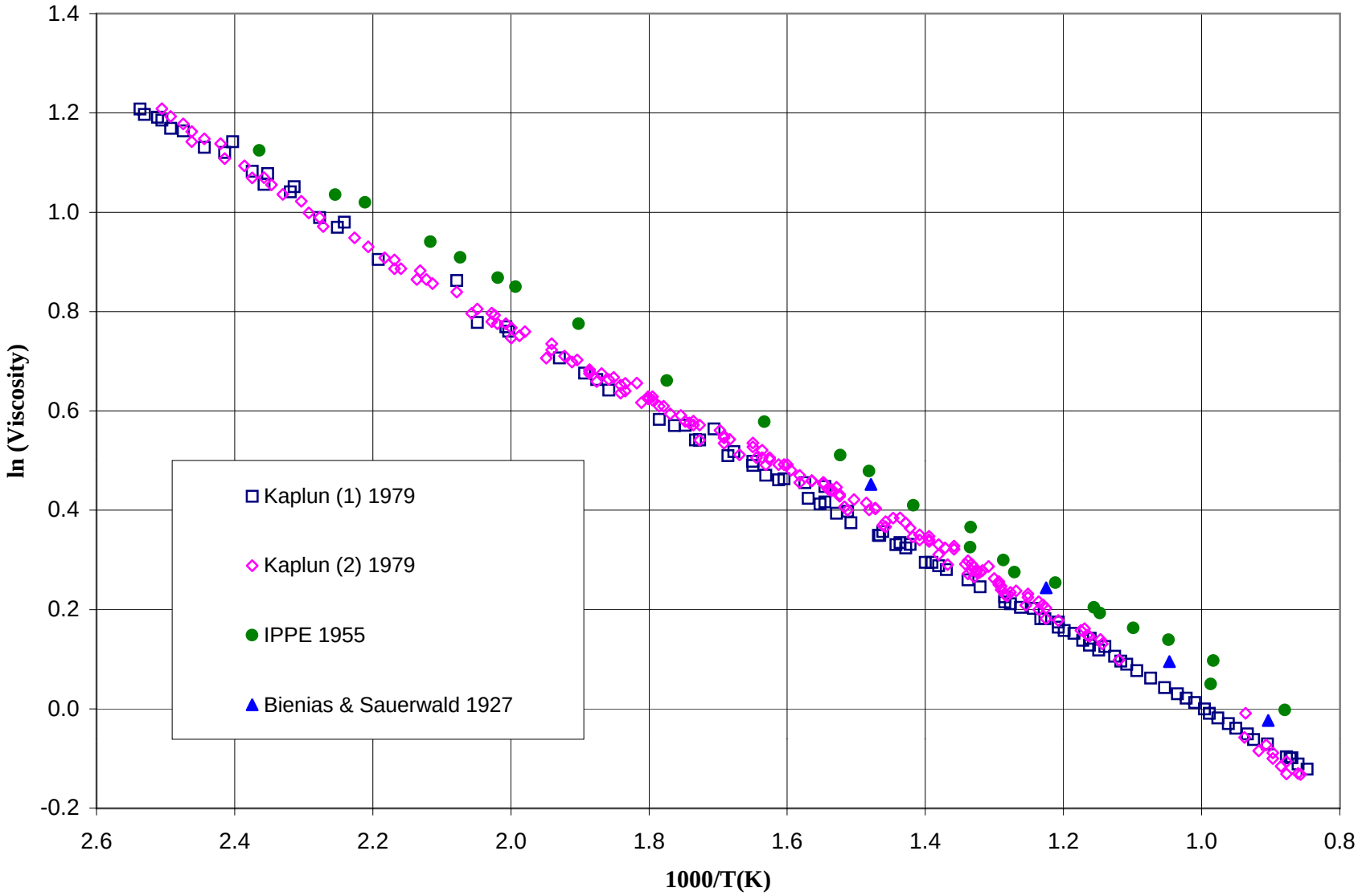

**Fig. 5 Pb-Bi Eutectic DynamicViscosity**

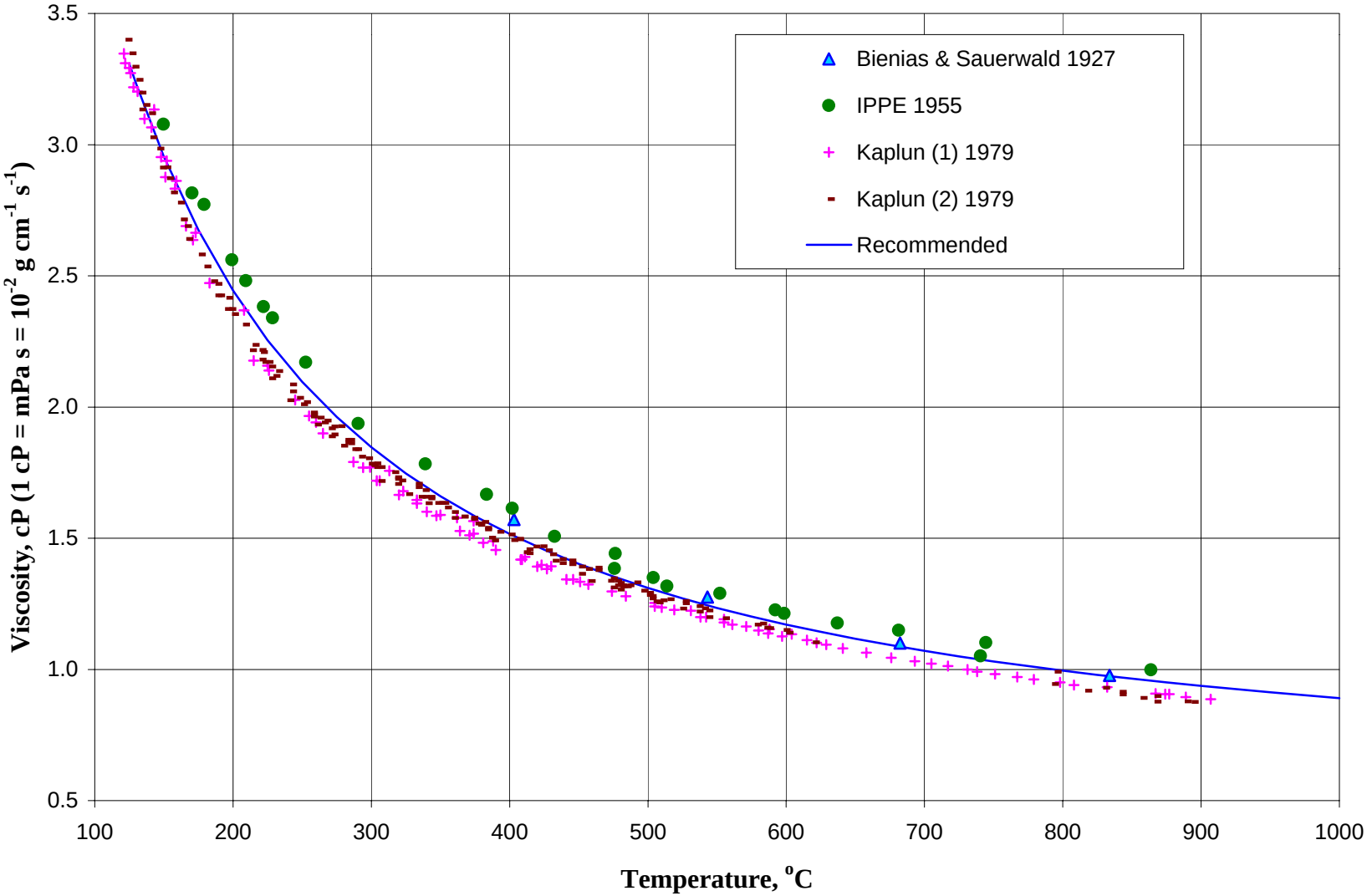

**Fig. 6 Comparison of Pb-Bi Eutectic DynamicViscosity Equations**

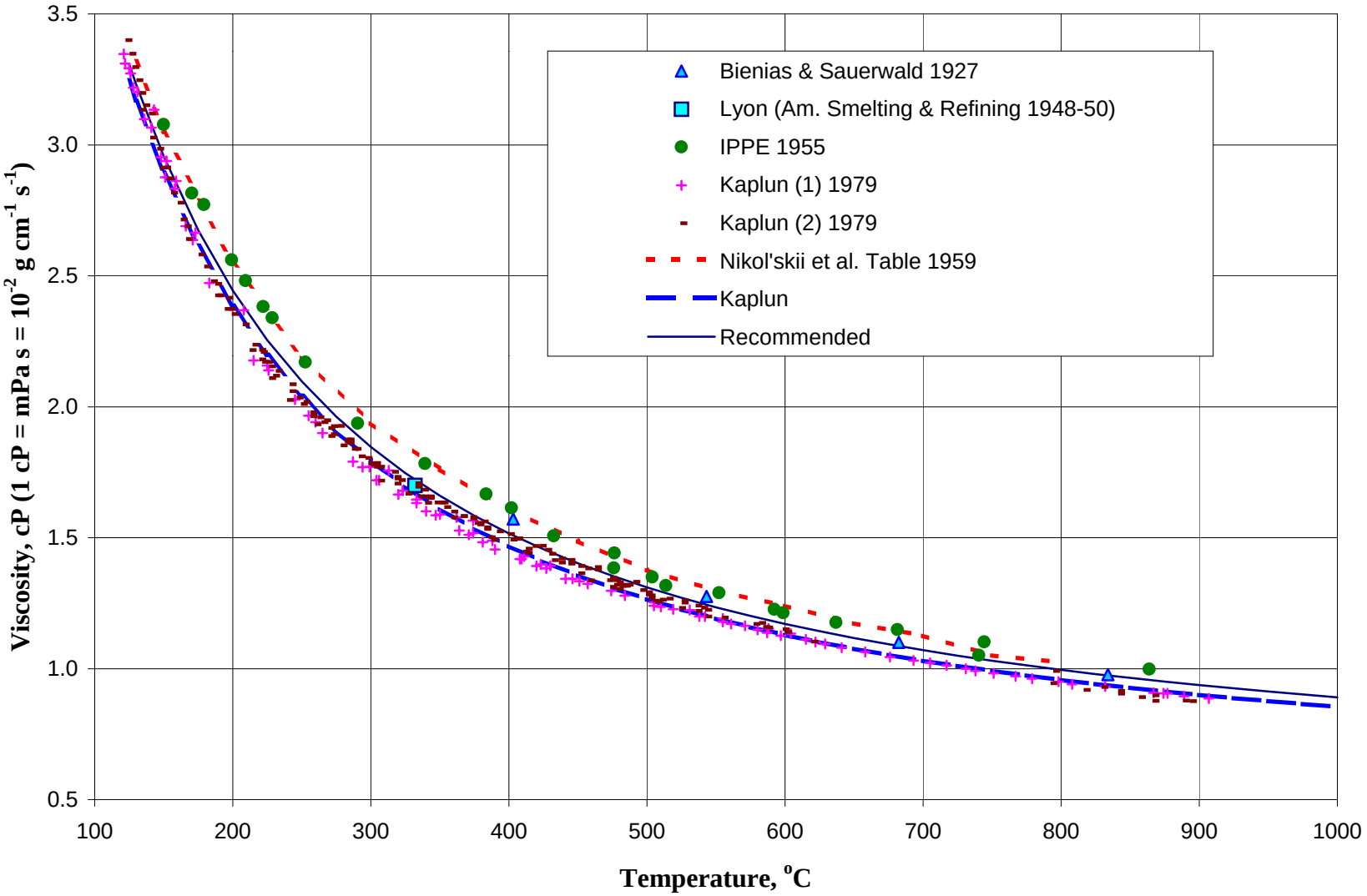

**Fig. 7 Comparison of Pb-Bi Eutectic Viscosity with Athena Equation**

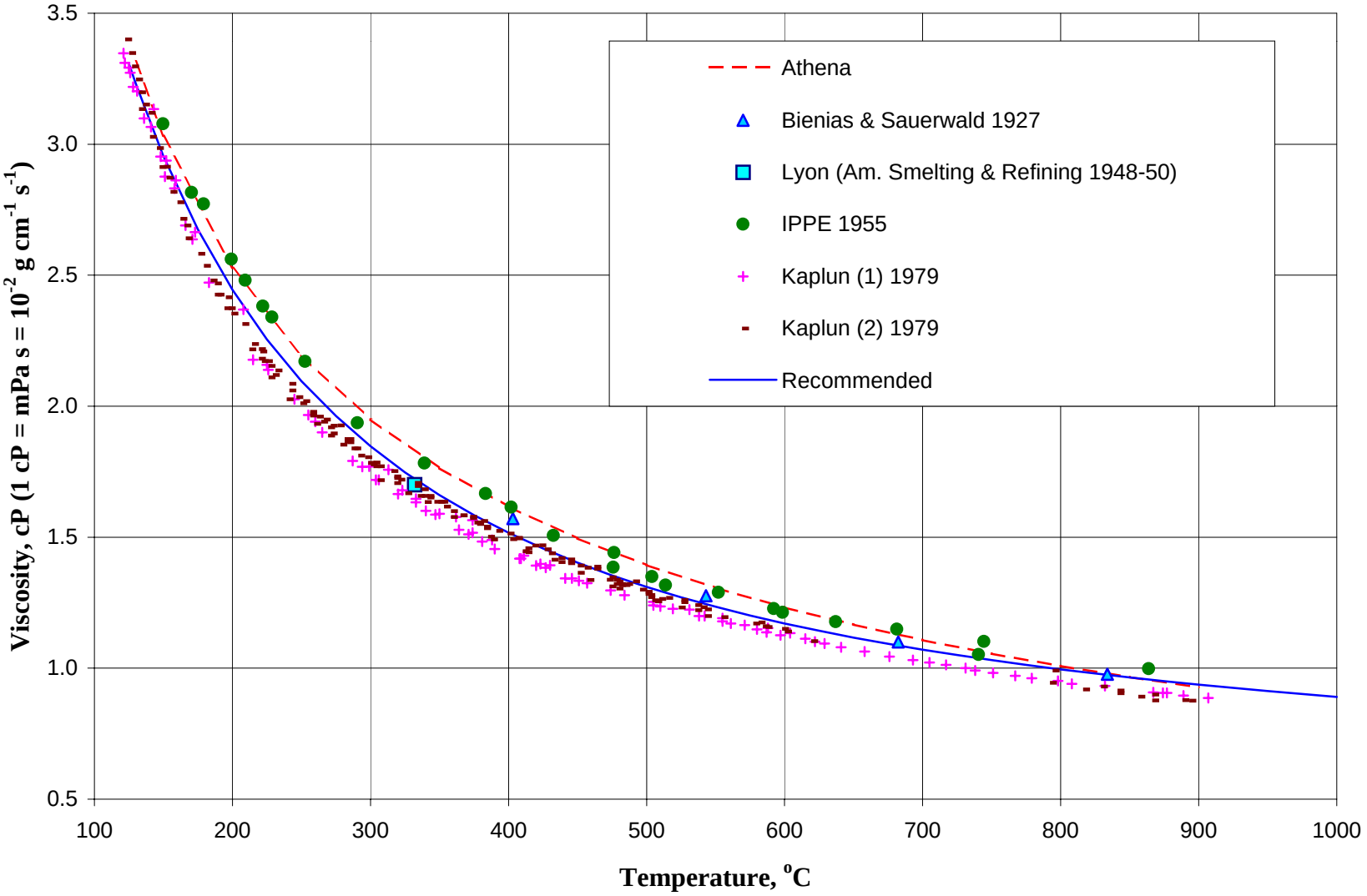

**Fig. 8 Recommended & ATHENA Equations for Pb-Bi Eutectic Viscosity**