

1.4 COMPRESSIBILITY AND SPEED OF SOUND

1.4.1 ADIABATIC COMPRESSIBILITY

Summary

Recommended values for the adiabatic compressibility of liquid sodium and sodium vapor are given in Table 1.4-1 and graphed, respectively, in Figs. 1.4-1 and 1.4-2. The dashed lines in the graphs represent the uncertainties in the recommended values. Estimated uncertainties as a function of temperature are given in Tables 1.4-2 and 1.4-3.

For liquid sodium, the recommended values for the adiabatic compressibility, β_S , in MPa^{-1} are calculated from

$$\beta_S = \beta_{S, m} \frac{\left(1 + \frac{\theta}{b}\right)}{(1 - \theta)}, \quad (1)$$

where the adiabatic compressibility at the melting point, $\beta_{S, m}$, is equal to

$$\beta_{S, m} = 1.717 \times 10^{-4} \text{ MPa}^{-1},$$

the constant b is

$$b = 3.2682,$$

and the parameter θ is defined by

$$\theta = \frac{(T - T_m)}{(T_C - T_m)}. \quad (2)$$

T_m and T_C are, respectively, the temperatures at the melting point (371 K) and critical point (2503.7 K).

The adiabatic compressibility of sodium vapor was calculated from the heat capacity at constant volume (C_V), the heat capacity at constant pressure (C_P), and the isothermal compressibility (β_T) using the thermodynamic relation

$$\beta_S = \beta_T \left(\frac{C_V}{C_P} \right). \quad (3)$$

Table 1.4-1 Adiabatic Compressibility of Sodium

Temperature (K)	Liquid $\beta_s \times 10^4$ (MPa ⁻¹)	Vapor β_s (MPa ⁻¹)
400	1.75	3.14×10^9
500	1.86	8.12×10^6
600	1.99	1.36×10^5
700	2.13	7.31×10^3
800	2.28	8.14×10^2
900	2.46	1.48×10^2
1000	2.66	3.78×10^1
1100	2.88	1.24×10^1
1200	3.14	4.90
1300	3.45	2.24
1400	3.81	1.14
1500	4.24	6.39×10^{-1}
1600	4.77	3.83×10^{-1}
1700	5.42	2.46×10^{-1}
1800	6.27	1.67×10^{-1}
1900	7.39	1.19×10^{-1}
2000	8.87	8.79×10^{-2}
2100	11.3	6.78×10^{-2}
2200	15.2	5.44×10^{-2}
2300	22.9	4.61×10^{-2}
2400	45.6	4.45×10^{-2}
2500	1291.31	3.74×10^{-1}

Discussion

Adiabatic Compressibility of Liquid Sodium — The parameter b in Eq. (1) for the adiabatic compressibility (β_s) of liquid sodium was obtained by fitting the adiabatic compressibilities from the melting point to 1773 K. The adiabatic compressibilities in this temperature range were calculated from the liquid density (ρ_l) and speed of sound in liquid sodium (v) using the relation

$$\beta_s = \frac{1}{\rho_l v^2} . \quad (3)$$

Table 1.4-2 Estimated Uncertainties in the Recommended Values for Adiabatic Compressibility of Liquid Sodium

Temperature (K)	β_S (MPa ⁻¹)	Uncertainty, $\left(\frac{\delta\beta_S}{\beta_S}\right)$ (%)
371 ≤ T ≤ 1400	$\beta_S = 1.717 \times 10^{-4} \frac{\left(1 + \frac{\theta}{b}\right)}{(1 - \theta)}$ <p>where $b = 3.2682$,</p> $\theta = \frac{T - T_m}{T_c - T_m},$ $T_m = 371 \text{ K},$ $T_c = 2503.7 \text{ K}.$	2
1400 < T ≤ 2000		25
2000 < T ≤ 2200		37
2200 < T ≤ 2400		49
2400 < T ≤ 2503		55

Table 1.4-3 Estimated Uncertainties in the Recommended Values for the Adiabatic Compressibility of Sodium Vapor

Temperature (K)	β_S (MPa ⁻¹)	Uncertainty, $\left(\frac{\delta\beta_S}{\beta_S}\right)$ (%)
371 ≤ T ≤ 500	$\beta_S = \beta_T \left(\frac{C_V}{C_P}\right)$	50
500 < T ≤ 1000		35
1000 < T ≤ 1600		30
1600 < T ≤ 2000		60
2000 < T ≤ 2200		80
2200 < T ≤ 2503.7		100

The liquid density (ρ_l) is given by

$$\rho_l = \rho_C + f \left(1 - \frac{T}{T_C} \right) + g \left(1 - \frac{T}{T_C} \right)^h, \quad (5)$$

where the parameters for density in $\text{kg}\cdot\text{m}^{-3}$ and temperature (T) in kelvins are

$$\begin{aligned} \rho_C &= 219., \\ f &= 275.32, \\ g &= 511.58, \\ h &= 0.5, \end{aligned}$$

and ρ_C , the density at the critical temperature, is $219 \text{ kg}\cdot\text{m}^{-3}$.

The recommended equation for the speed of sound (v) in $\text{m}\cdot\text{s}^{-1}$ is the quadratic equation determined by Fink and Leibowitz⁽¹⁾ who fit the available speed of sound data from the melting point to 1773 K; their equation is

$$v = 2660.7 - 0.37667 T - 9.0356 \times 10^{-5} T^2 \quad (6)$$

for 371 K \leq T \leq 1773 K .

In Fig. 1.4-3, recommended values for the adiabatic compressibility of liquid sodium are compared with values from assessments by Fink and Leibowitz,⁽¹⁾ Hornung,⁽²⁾ and Bystrov et al.⁽³⁾ Equations of similar form to Eqs. (1-6) were used by Fink and Leibowitz and by Hornung. Bystrov et al. used a linear equation to represent the speed of sound in sodium and a seven-term polynomial to represent the liquid density. They used Eq. (4) to calculate the adiabatic compressibility in the range of experimental data and to extrapolate to high temperatures. Values from all three assessments are in excellent agreement (within 2%) through 1600 K. Disagreement between the values calculated by Bystrov et al. and values from other assessments increases with increasing temperature above 1600 K. Deviations of values calculated in these assessments from the recommended values are shown in Fig. 1.4-4. The deviations shown in the graph in Fig. 1.4-4 were defined as

$$Deviations = \left(\frac{[\beta_S(Other) - \beta_S(Recommended)] 100\%}{\beta_S(Recommended)} \right) . \quad (7)$$

The increasing deviation with temperature of values calculated by Bystrov et al. arises from their choice of equations for the density and the adiabatic compressibility which do not have the correct physical behavior at the critical point. Extrapolation of these equations beyond the range of experimental data leads to large differences. Values calculated by Bystrov et al. are low by 63% at 2400 K and by 98% at 2500 K. Values calculated by Hornung and by Fink and Leibowitz are within 2% of the recommended values through 2000 K, the highest temperature calculated by Hornung. At 2400 K, values calculated by Fink and Leibowitz deviate by -5%. The magnitude of the deviation increases as the critical point is reached because of their selection of a different critical temperature and critical density than the one recommended in this assessment.

Adiabatic Compressibility of Sodium Vapor — The adiabatic compressibility of sodium vapor was calculated from the isothermal compressibility (β_T) and the heat capacities at constant pressure (C_p) and constant volume (C_v) using the thermodynamic relation given in Eq. (3). These thermodynamic properties are defined below in Eqs. (8-10). The isothermal compressibility (β_T) of sodium vapor was calculated from the instantaneous volumetric thermal-expansion coefficient (α_p) and the thermal-pressure coefficient (γ_v) using the thermodynamic relation

$$\beta_T = \frac{\alpha_p}{\gamma_v} . \quad (8)$$

The heat capacity at constant pressure of sodium vapor has been calculated from the heat capacity along the saturation curve (C_σ) using the thermodynamic relations

$$C_p = C_\sigma + \left(\frac{T\alpha_p\gamma_\sigma}{\rho_g} \right) , \quad (9)$$

where α_p is the thermal-expansion coefficient, γ_σ is the partial derivative of the pressure with respect to temperature along the saturation curve, and ρ_g is the vapor density. The heat capacity at constant volume of saturated sodium vapor was calculated using the thermodynamic relation

$$C_V = C_P - \left(\frac{T \alpha_p \gamma_V}{\rho_g} \right), \quad (10)$$

where C_P , α_p , ρ_g , and γ_V are, respectively, the heat capacity at constant pressure, the instantaneous volumetric thermal-expansion coefficient, the density of sodium vapor and the thermal-pressure coefficient.

The instantaneous volumetric thermal-expansion coefficient for sodium vapor $(\alpha_p)_g$ was calculated from the relation

$$(\alpha_p)_g = \frac{(\alpha_\sigma)_g}{\left(1 - \frac{\gamma_\sigma}{\gamma_V} \right)}, \quad (11)$$

where the coefficient of thermal expansion along the saturation curve $(\alpha_\sigma)_g$ for sodium vapor is defined as

$$(\alpha_\sigma)_g = - \frac{1}{\rho_g} \left(\frac{\partial \rho_g}{\partial T} \right)_\sigma.$$

The vapor density (ρ_g) was calculated from the enthalpy of vaporization (ΔH_g) , the temperature derivative of the vapor pressure (γ_σ) , and the liquid density (ρ_l) using the relation

$$\rho_g = \left(\frac{\Delta H_g}{T \gamma_\sigma} + \frac{1}{\rho_l} \right)^{-1}, \quad (13)$$

where

$$\gamma_\sigma = \left(\frac{\partial P}{\partial T} \right)_\sigma. \quad (14)$$

The vapor pressure, P , is given by an equation derived by Browning and Potter:⁽⁴⁾

$$\ln P = a + \frac{b}{T} + c \ln T . \quad (15)$$

Then γ_σ , the temperature derivative of the pressure along the saturation curve, is

$$\gamma_\sigma = \left(-\frac{b}{T^2} + \frac{c}{T} \right) \exp\left(a + \frac{b}{T} + c \ln T \right) , \quad (16)$$

and the coefficients in Eqs. (15, 16) for P in MPa and T in kelvins are defined as

$$\begin{aligned} a &= 11.9463, \\ b &= -12633.73, \\ c &= -0.4672. \end{aligned}$$

The enthalpy of vaporization, ΔH_g , in $\text{kJ}\cdot\text{kg}^{-1}$, is given by

$$\Delta H_g = 393.37 \left(1 - \frac{T}{T_c} \right) + 4398.6 \left(1 - \frac{T}{T_c} \right)^{0.29302} \quad (17)$$

$$\text{for } 371 \text{ K} \leq T \leq 2503.7 \text{ K} ,$$

where T_c is the critical temperature, 2503.7 K, and T is the temperature in kelvins. The liquid density (ρ_l) used in the calculation of the vapor density in Eq. (13) is defined in Eq. (5).

The thermal-pressure coefficient (γ_v) used in Eqs. (8, 10, 11) is defined below in Eqs. (18, 19). Below 1600 K, the thermal-pressure coefficient was calculated using the quasi-chemical approximation.⁽⁵⁾ Values calculated via the quasi-chemical approximation were fit to an equation so that a functional form is available for calculation of all the vapor properties. This equation for γ_v in $\text{MPa}\cdot\text{K}^{-1}$ is

$$\gamma_V = \left(-\frac{b}{T^2} + \frac{c}{T} + d + 2eT \right) \exp\left(a + \frac{b}{T} + c \ln T + dT + eT^2 \right) \quad (18)$$

for $371 \text{ K} \leq T \leq 1600 \text{ K}$,

where

$$\begin{aligned} a &= 8.35307, \\ b &= -12905.6, \\ c &= -0.45824, \\ d &= 2.0949 \times 10^{-3}, \\ e &= -5.0786 \times 10^{-7}. \end{aligned}$$

At the critical point, the thermal-pressure coefficient (γ_V) must equal γ_σ , the slope of the vapor pressure curve. Above 1600 K, the thermal-pressure coefficient was extrapolated to the critical point using the same form of equation used by Fink and Leibowitz:⁽¹⁾

$$\gamma_V = \gamma_V^C + A \left(1 - \frac{T}{T_C} \right)^{\frac{1}{2}} + B \left(1 - \frac{T}{T_C} \right) \quad (19)$$

for $1600 \text{ K} \leq T \leq 2500 \text{ K}$,

where

$$\begin{aligned} \gamma_V^C &= \gamma_\sigma^C = 4.6893 \times 10^{-2}, \\ A &= -2.5696 \times 10^{-3}, \\ B &= 3.5628 \times 10^{-5}, \\ T_C &= 2503.7 \text{ K}. \end{aligned}$$

The superscript *C* and subscript *C* in Eq. (19) denote the value at the critical temperature (T_C). The parameters *A* and *B* in Eq. (19) were determined by matching the value and temperature derivative of the thermal-pressure coefficient at 1600 K. The equation fitting the thermal-pressure coefficient below 1600 K and the extrapolation to the critical point are shown in Fig. 1.4-5. The derivative of the vapor pressure, γ_σ , has been included in the figure.

The heat capacity at constant pressure, C_p , defined in Eq. (9), is a function of the heat capacity along the saturation curve. The heat capacity along the saturation curve, C_σ , is defined as

$$C_\sigma = T \left(\frac{\partial S}{\partial T} \right)_\sigma . \quad (20)$$

It is related to the partial derivative of the enthalpy along the saturation curve by

$$C_\sigma = \left(\frac{\partial H}{\partial T} \right)_\sigma - \frac{\gamma_\sigma}{\rho_g} ,$$

where the enthalpy of the vapor along the saturation curve is the sum of the enthalpy of liquid sodium on the saturation curve and the enthalpy of vaporization;

$$H(g, T) - H(s, 298) = H(AVG, T) - H(s, 298) + \Delta H_g . \quad (22)$$

The enthalpy of vaporization, ΔH_g , is given in Eq. (17). Below 2000 K, the liquid enthalpy in $\text{kJ}\cdot\text{kg}^{-1}$, was calculated from the CODATA equation⁽⁶⁾ given by Cordfunke and Konings:⁽⁷⁾

$$\begin{aligned} H(AVG, T) - H(s, 298.15) = & - 365.77 + 1.6582 T - 4.2395 \times 10^{-4} T^2 \\ & + 1.4847 \times 10^{-7} T^3 + 2992.6 T^{-1} . \end{aligned} \quad (23)$$

Above 2000 K, the law of rectilinear diameters was used to extrapolate the average of the liquid and vapor enthalpies to the critical point. The enthalpy of sodium vapor relative to the solid at 298.15 K is the average enthalpy plus one half the enthalpy of vaporization. In $\text{kJ}\cdot\text{kg}^{-1}$, the average enthalpy is given by

$$H(AVG, T) - H(s, 298.15) = E + FT \quad (24)$$

$$\text{for } 2000 \text{ K} \leq T \leq 2503.7 \text{ K} ,$$

where

$$E = 2128.4 ,$$

$$F = 0.86496 .$$

Thus, the enthalpy of sodium vapor is

$$H(g, T) - H(s, 298.15) = E + FT + \frac{1}{2} \Delta H \quad (25)$$

for $2000 \text{ K} \leq T \leq 2503.7 \text{ K}$.

Values for the adiabatic compressibility of sodium vapor were available only in the assessment by Fink and Leibowitz.⁽¹⁾ Values from their assessment are compared with recommended values in Fig. 1.4-6. Deviations of values given by Fink and Leibowitz from values recommended in this assessment, defined as in Eq. (7), are within 10% for most of the temperature range, as shown in Fig. 1.4-7. Large deviations occur at both low and high temperatures. The low temperature deviations are due to differences in the thermal-pressure coefficient and enthalpy of vaporization at the low temperatures. Fink and Leibowitz calculated the thermal-pressure coefficient and the enthalpy of vaporization from the quasi-chemical approximation below 1600 K. In this assessment, values from the quasi-chemical approximation were fit with equations to provide mathematical functions for calculation for the entire temperature range. These differences led to differences near the melting point. The 25% deviation at 2400 K arises from differences in dependent parameters as the critical temperature is approached. Different values were chosen for the critical temperature in the two assessments. Fink and Leibowitz used 2509.4 K, whereas 2503.7 K has been selected for the critical temperature in this assessment.

Although values for the adiabatic and isothermal compressibilities of sodium vapor are not available from the assessment by Bystrov et al.,⁽³⁾ the ratio of the vapor heat capacities, which is related to the ratio of the vapor compressibilities, is given in their assessment. The thermodynamic relation between these two ratios is

$$\frac{\beta_T}{\beta_S} = \frac{C_P}{C_V} . \quad (26)$$

In Fig. 1.4-8, values for this ratio from the assessment of Fink and Leibowitz,⁽¹⁾ the assessment by Bystrov et al.,⁽²⁾ and this recommendation are compared. Percent deviations of the ratios from these assessments relative to the ratios calculated from the recommended values are shown in Fig. 1.4-9. For most of the temperature range, deviations are within 4%. Higher deviations were found with respect to the ratios from Fink and Leibowitz at 400 K and above 2400 K. At 2000 K, the ratio given by Bystrov et al. deviates by about 9%.

Uncertainty

The uncertainties in the recommended values for the adiabatic compressibilities of liquid sodium and sodium vapor have been calculated from uncertainties in the dependent parameters assuming that all uncertainties are independent. If x_i are the dependent parameters, the square of the uncertainty in the calculated quantity $(\delta\beta_s)$ is given by

$$(\delta\beta_s)^2 = \sum \left(\frac{\partial\beta_s}{\partial x_i} \right)^2 (\delta x_i)^2, \quad (27)$$

where δx_i are the uncertainties in the dependent parameters.

Adiabatic Compressibility of Liquid Sodium — Differentiating Eq. (4) for the adiabatic compressibility of liquid sodium leads to Eq. (28) for the uncertainty

$$\delta\beta_s = \sqrt{(\delta\rho_l)^2 + 4(\delta v)^2}, \quad (28)$$

where $(\delta\rho_l)$ is the uncertainty in the liquid density and (δv) is the uncertainty in the speed of sound. Calculated uncertainties in the adiabatic compressibility of liquid sodium are 2% from 371 through 1400 K. They increase to 25% at 2000 K and to 55% at 2500 K. Calculated uncertainties are tabulated as a function of temperature in Table 1.4-2. Comparison of calculated uncertainties with deviations of other assessments from the recommended values (graphed in Fig. 1.4-4) shows that values from Fink and Leibowitz⁽¹⁾ and from Hornung⁽²⁾ are within 2% of the recommended values for the entire temperature range. However, values calculated by Bystrov et al.⁽³⁾ deviate by -10% at 1900 K and by -98% at 2500 K. These large deviations at the higher temperatures are consistent with the high calculated uncertainties. They arise from the use of polynomial equations for extrapolation of density and speed of sound to the critical temperature.

Adiabatic Compressibility of Sodium Vapor — Because the functional forms of the thermodynamic properties used to calculate the adiabatic compressibility of sodium vapor are so complex and these properties are not independent, the square of the uncertainties cannot be calculated from the square of the uncertainties of the dependent parameters. Consequently, the uncertainties in the calculated adiabatic compressibilities of sodium vapor were calculated from

the uncertainties in the fundamental properties used to calculate the dependent variables in Eq. (3). Uncertainties in the adiabatic compressibility of sodium vapor were calculated from the uncertainties in the thermal-pressure coefficient, the vapor enthalpy, and the enthalpy of vaporization using the approximation

$$\delta\beta_s = \sqrt{(\delta\gamma_v)^2 + (\delta H_g)^2 + (\delta\Delta H)^2} \quad . \quad (29)$$

Calculated uncertainties have been included as dashed lines in Fig. 1.4-2 and are tabulated as a function of temperature in Table 1.4-3. The maximum uncertainty in each temperature range has been included in the table. High uncertainties (50%) are calculated below 500 K. These are consistent with the large deviation between values calculated by Fink and Leibowitz and by the recommended equations (see Fig. 1.4-7). Uncertainties decrease to 35% from 500 to 1000 K, and 30% from 1000 to 1600 K. Above 1600 K, they increase with temperature. These estimated uncertainties are consistent with the estimated uncertainties given by Fink and Leibowitz.⁽¹⁾ However, they are considerably higher than deviations between values calculated by Fink and Leibowitz and by the recommended equations. These deviations are on the order of 3% for the 800 to 2100 K temperature range. They increase to 25% at 2400 K. Although no other values of adiabatic compressibility of the vapor are available for comparison, comparisons with ratios of isothermal compressibility to adiabatic compressibilities of Bystrov et al.⁽³⁾ indicate deviations are on the order of 3% for temperatures between 800 and 1900 K. At 2000 K, the deviation was -8.7%. Thus, uncertainties estimated using Eq. (29) appear to be conservative.

1.4.2 ISOTHERMAL COMPRESSIBILITY

Summary

Recommended values for the isothermal compressibility of liquid sodium and sodium vapor are given in Table 1.4-4 and graphed, respectively, in Figs. 1.4-10 and 1.4-11. The dashed lines in the graphs represent the uncertainties in the recommended values. Estimated uncertainties as a function of temperature are given in Tables 1.4-5 and 1.4-6.

Table 1.4-4 Isothermal Compressibility of Sodium

Temperature (K)	Liquid $\beta_T \times 10^4$ (MPa ⁻¹)	Vapor β_T (MPa ⁻¹)
400	1.93	5.56×10^9
500	2.12	1.12×10^7
600	2.34	1.82×10^5
700	2.60	9.75×10^3
800	2.89	1.10×10^3
900	3.23	2.03×10^2
1000	3.64	5.32×10^1
1100	4.11	1.79×10^1
1200	4.66	7.30
1300	5.33	3.43
1400	6.15	1.81
1500	7.16	1.04
1600	8.44	6.48×10^{-1}
1700	10.1	4.29×10^{-1}
1800	12.4	3.03×10^{-1}
1900	15.7	2.26×10^{-1}
2000	20.6	1.78×10^{-1}
2100	28.4	1.50×10^{-1}
2200	42.1	1.36×10^{-1}
2300	70.8	1.41×10^{-1}
2400	156.	1.99×10^{-1}
2500	547.	8.88

Table 1.4-5 Estimated Uncertainties in the Recommended Values for Isothermal Compressibility of Liquid Sodium

Temperature (K)	β_T (MPa ⁻¹)	Uncertainty, $\left(\frac{\delta\beta_T}{\beta_T}\right)$ (%)
$371 \leq T \leq 500$	$\beta_T = \frac{\beta_S C_\sigma + \left(\frac{T}{\rho_l}\right) \alpha_\sigma (\alpha_\sigma + \beta_S \gamma_\sigma)}{C_\sigma - \left(\frac{T}{\rho_l}\right) \gamma_\sigma (\alpha_\sigma + \beta_S \gamma_\sigma)}$	50
$500 < T \leq 1000$		35
$1000 < T \leq 1600$		30
$1600 < T \leq 2000$		60
$2000 < T \leq 2200$		80
$2200 < T \leq 2503.7$		100

Table 1.4-6 Estimated Uncertainties in the Recommended Values for the Isothermal Compressibility of Sodium Vapor

Temperature (K)	β_T (MPa ⁻¹)	Uncertainty, $\left(\frac{\delta\beta_T}{\beta_T}\right)$ (%)
$371 \leq T \leq 500$	$\beta_T = \frac{\alpha_p}{\gamma_v}$	50
$500 < T \leq 1600$		15
$1600 < T \leq 2000$		30
$2000 < T \leq 2200$		40
$2200 < T \leq 2400$		50
$2400 < T \leq 2503.7$		100

The isothermal compressibility (β_T) for liquid sodium was calculated from the thermodynamic relation

$$\beta_T = \frac{\beta_S C_\sigma + \left(\frac{T}{\rho_l}\right) \alpha_\sigma (\alpha_\sigma + \beta_S \gamma_\sigma)}{C_\sigma - \left(\frac{T}{\rho_l}\right) \gamma_\sigma (\alpha_\sigma + \beta_S \gamma_\sigma)}, \quad (30)$$

where β_S is the adiabatic compressibility, C_σ is the heat capacity along the saturation curve, α_σ is the thermal-expansion coefficient along the saturation curve, γ_σ is the temperature derivative of the vapor pressure along the saturation curve, and ρ_l is the liquid density.

The isothermal compressibility of sodium vapor was calculated from the instantaneous volumetric thermal-expansion coefficient of sodium vapor (α_p) and the thermal-pressure coefficient (γ_v) using the thermodynamic relation

$$\beta_T = \left(\frac{\alpha_p}{\gamma_v}\right). \quad (31)$$

Discussion

Isothermal Compressibility of Liquid Sodium — The isothermal compressibility of liquid sodium was calculated from the thermodynamic relation given in Eq. (30). The adiabatic compressibility (β_S) used in Eq. (30) is defined in Eqs. (1, 2, 4). The liquid density (ρ_l) is given in Eq. (5). The temperature derivative of the vapor pressure along the saturation curve (γ_σ) is given in Eq. (16). The thermal-expansion coefficient along the saturation curve (α_σ) is defined in terms of the liquid density in Eq. (32):

$$\alpha_\sigma = -\frac{1}{\rho_l} \left(\frac{\partial \rho_l}{\partial T}\right)_\sigma. \quad (32)$$

The heat capacity along the saturation curve (C_σ) is related to the partial derivative with respect to temperature of the liquid enthalpy increment along the saturation curve according to the thermodynamic relation

$$C_{\sigma} = \left(\frac{\partial H}{\partial T} \right)_{\sigma} - \frac{\gamma_{\sigma}}{\rho_l} . \quad (33)$$

Equation (23) is the recommended equation for the liquid enthalpy increment along the saturation curve below 2000 K. Above 2000 K, the liquid enthalpy is calculated from the average enthalpy (Eq. [24]) minus one half the enthalpy of vaporization (Eq. [17]); i.e.,

$$H(AVG, T) - H(s, 298.15) = E + FT - \frac{1}{2} \Delta H_g \quad (34)$$

for $2000 \text{ K} \leq T \leq 2503.7 \text{ K}$,

where

$$E = 2128.4 ,$$

$$F = 0.86496 .$$

Isothermal compressibilities for liquid sodium have been calculated by Bystrov et al.,⁽³⁾ Hornung,⁽²⁾ and Fink and Leibowitz.⁽¹⁾ In Fig. 1.4-12, values from these assessments are compared with recommended values. There is good agreement between recommendations from all assessments through 1300 K. Deviations of recommended values from values from other assessments defined as

$$Deviations = \left(\frac{[\beta_T(Other) - \beta_T(Recommended)] 100\%}{\beta_T(Recommended)} \right) , \quad (35)$$

are shown in Fig. 1.4-13. Above 1500 K, values from the other assessments are lower than the recommended values. From 1200 to 2400 K, values calculated by Bystrov et al. are closest to the recommended values. This may be because both calculations used the CODATA equation for the enthalpy of liquid sodium up to 2000 K.

Isothermal Compressibility of Sodium Vapor — The isothermal compressibility of sodium vapor was calculated from the instantaneous volumetric thermal-expansion coefficient for the vapor and the thermal-pressure coefficient using the thermodynamic relation given in Eq. (31). The vapor instantaneous volumetric thermal-expansion coefficient is defined in Eqs. (11-17). The thermal-pressure coefficient is given in Eqs. (18, 19).

In Fig. 1.4-14, the recommended values for the isothermal compressibility of sodium vapor are compared with values calculated by Fink and Leibowitz.⁽¹⁾ Values calculated by Fink and Leibowitz are higher than recommended values at all temperatures. Deviations defined in accord with Eq. (35) are shown in Fig. 1.4-15. Highest percent deviations are at the high and low temperatures. From 700 through 2100 K, deviations between these two calculations are within 6%. The high percent deviations at low temperatures arise from differences in the thermal-pressure coefficient in the two calculations, as discussed above.

Uncertainty

The uncertainties in the recommended values for the isothermal compressibilities of liquid sodium and sodium vapor have been calculated from uncertainties in the dependent parameters. If the dependent parameters (χ_i) are independent of each other, then the square of the uncertainty in the calculated quantity ($\delta\beta_T$) is given by

$$(\delta\beta_T)^2 = \sum \left(\frac{\partial\beta_T}{\partial\chi_i} \right)^2 (\delta\chi_i)^2, \quad (36)$$

where $\delta\chi_i$ are the uncertainties in the dependent parameters.

Isothermal Compressibility of Liquid Sodium — The isothermal compressibility, calculated using Eq. (30), is a function of thermodynamic variables which are functions of some of the same variables. For example, liquid density enters Eq. (30) not only explicitly but also through the adiabatic compressibility, the thermal-expansion coefficient along the saturation curve, and the heat capacity along the saturation curve. Vapor pressure enters both through the derivative along the saturation curve and through the heat capacity along the saturation curve. So the assumptions for application of Eq. (36) are not valid. Because of the complicated relationships between the dependent parameters, the uncertainty in the isothermal compressibility of liquid sodium has been approximated by

$$\delta\beta_T = \sqrt{(\delta\rho_l)^2 + (\delta\beta_s)^2 + (\delta C_o)^2}, \quad (37)$$

where $(\delta\rho_l)$ is the uncertainty in the liquid density, $(\delta\beta_s)$ is the uncertainty in the adiabatic compressibility, and (δC_o) is the uncertainty in the heat capacity along the saturation curve calculated from

$$\delta C_{\sigma} = \sqrt{4(\delta H_f)^2 + 4(\delta P)^2 + (\delta \rho_f)^2} . \quad (38)$$

The uncertainties calculated using Eqs. (37, 38) do not differ significantly from uncertainties calculated using other approximations such as calculation of uncertainties from the basic parameters (H , ΔH_g , P , v , ρ_f). Calculated uncertainties in the isothermal compressibility of liquid sodium are shown in Table 1.4-5. They range from 3% from 371 through 1000 K to 65% above 2400 K. Comparison of these estimated uncertainties with deviations of other assessments from the recommended values for the isothermal compressibility shows that the deviations are within the estimated uncertainties for most of the temperature range. Deviations are on the order of 3% or less from 371 through 1000 K. Between 1000 and 1600 K, all deviations are within 6% except for those from Fink and Leibowitz.⁽¹⁾ The percent deviation for values calculated by Fink and Leibowitz are 10% at 1500 K and 13% at 1600 K. The 30% deviation of values from Fink and Leibowitz are included in the 30% uncertainty from 1600 to 2000 K. The 45% and 60% uncertainties for the temperature ranges 2000 to 2200 K and 2200 to 2400 K are greater than any deviations between calculated values in the different assessments. However, deviations at 2500 K are greater than the 65% uncertainty given for temperatures greater than 2400 K. This is because the isothermal compressibility becomes very large as the critical point is approached and different values are selected for the critical temperature in the different assessments.

Isothermal Compressibility of Sodium Vapor— The uncertainties in the recommended values for the isothermal compressibility of sodium vapor were estimated using Eq. (36) and the thermodynamic relation for isothermal compressibility of sodium vapor given in Eq. (8). The uncertainties were estimated from the uncertainties in the instantaneous volumetric thermal-expansion coefficient for sodium vapor and the thermal-pressure coefficient according to

$$\delta \beta_T = \sqrt{(\delta \alpha_P)^2 + (\delta \gamma_V)^2} . \quad (39)$$

Uncertainties calculated with Eq. (39) are given for each temperature range in Table 1.4-6.

As for the adiabatic compressibilities, a high percent uncertainty (50%) is calculated for low temperatures (371 to 500 K). This is consistent with the large deviations between values from this calculation and values from Fink and Leibowitz⁽¹⁾ at low temperatures. At 400 K, the

deviation is 33%. The large percent deviation arises from the use of an equation to represent the thermal-pressure coefficient and the enthalpy of vaporization at low temperatures rather than use values from the quasi-chemical approximation. Uncertainties are 15% from 500 to 1600 K. Then they increase with increasing temperature to 100% at 2503 K. Calculated deviations between values given by Fink and Leibowitz and recommended values lie within the estimated uncertainties.

1.4.3 SPEED OF SOUND

Summary

Below 1773 K, the speed of sound (v) in liquid sodium in $\text{m}\cdot\text{s}^{-1}$ is given by the quadratic equation determined by Fink and Leibowitz⁽¹⁾ from fitting the data from 370 to 1270 K of Leibowitz et al.⁽⁸⁾ and data from 1010 to 1770 K from Chasanov et al.⁽⁹⁾

$$(4) \quad v = 2660.7 - 0.37667 T - 9.0356 \times 10^{-5} T^2 \quad (40)$$

for 371 K \leq T \leq 1773 K .

Above 1773 K, the speed of sound in liquid sodium is calculated from the liquid adiabatic compressibility (β_s) and the liquid density (ρ_l) using the thermodynamic relation

$$v = \frac{1}{\sqrt{\rho_l \beta_s}} \quad (41)$$

for 1773 K < T \leq 2503.7 K .

Recommended values for the speed of sound in liquid sodium are given in Table 1.4-7 and shown in Fig. 1.4-16. Estimated uncertainties in the recommended values have been included as dotted lines in Fig. 1.4-16 and are given in Table 1.4-8.

Discussion

The liquid density of sodium is given in Eq. (5). The adiabatic compressibility of liquid sodium is defined in Eqs. (1, 2). In Fig. 1.4-17, values for the speed of sound calculated from Eqs. (40, 41) are compared with values calculated by Fink and Leibowitz,⁽¹⁾ Bystrov et al.,⁽³⁾ and the extrapolation of the quadratic equation (Eq. [40]) to the critical point. This

Table 1.4-7 Speed of Sound in Liquid Sodium

Temperature (K)	Speed of Sound (m · s⁻¹)
400	2496
500	2450
600	2402
700	2353
800	2302
900	2249
1000	2194
1100	2137
1200	2079
1300	2018
1400	1956
1500	1892
1600	1827
1700	1759
1800	1676
1900	1587
2000	1487
2100	1372
2200	1235
2300	1060
2400	810
2500	180

Table 1.4-8 Estimated Uncertainties in the Recommended Values for Speed of Sound in Liquid Sodium

Temperature (K)	v ($\text{m} \cdot \text{s}^{-1}$)	Uncertainty, $\left(\frac{\delta_v}{v}\right)$ (%)
$371 \leq T \leq 1600$	$v = 2660.7 - 0.37667 T - 9.0356 \times 10^{-5} T^2$	1
$1600 < T \leq 1773^{(a)}$		1.3 ^(a)
$1773 < T \leq 2000^{(a)}$	$v = \frac{1}{\sqrt{\beta_s \rho_l}}$	10 ^(a)
$2000 < T \leq 2200^{(a)}$		16 ^(a)
$2200 < T \leq 2400^{(a)}$		22 ^(a)
$2400 < T \leq 2503^{(a)}$		25 ^(a)

^(a)From 1700 to 2503.7 K, the uncertainty is approximated by $\frac{\delta v}{v}$ (%) = $-48 + 0.029 T$

extrapolation is labeled "quad-ext" in the figure. Bystrov et al. represent the speed of sound with a linear equation, which they extrapolate to the critical point. Extrapolation of these polynomial representations of the speed of sound to the critical point do not give proper physical behavior for the speed of sound or for the adiabatic compressibility (calculated from the speed of sound) at the critical point. The extrapolation used here is identical to that used by Fink and Leibowitz.⁽¹⁾ It gives proper physical behavior at the critical point.

Deviations between recommended values and those of Bystrov et al. and Fink and Leibowitz, defined as

$$Deviations = \left(\frac{[v(Other) - v(Recommended)] 100\%}{v(Recommended)} \right), \quad (42)$$

are shown in Fig. 1.4-18. Values given by Fink and Leibowitz and those recommended here are identical through 1773 K. Above 1773 K, deviations increase with temperature due to differences in the density and critical temperature in the two assessments. Values given by Bystrov et al. agree within 1.3% through 1700 K. Above 1700 K, deviations increase with temperature, as shown in the figure. At 2400 K, deviations are 71%.

Uncertainty

Uncertainties in the recommended values for the speed of sound have been estimated from the deviations of the various assessments and the uncertainties given in these assessments. From 371 through 1600 K, the uncertainty is estimated as 1%. All recommended values are within this uncertainty in this temperature range. Above 1600 K, uncertainties increase with temperature according to the equation

$$\frac{\delta v}{v}(\%) = -48 + 0.029 T . \quad (43)$$

At 2500 K, uncertainties are estimated as 25%. This uncertainty is lower than the deviation between recommended values and those given by Bystrov et al. Because the equation used by Bystrov et al. does not have the proper behavior near the critical point, larger deviations than estimated uncertainties may be expected.

REFERENCES

1. J. K. Fink and L. Leibowitz, *Thermophysical Properties of Sodium*, **ANL-CEN-RSD-79-1**, Argonne National Laboratory Report (May 1979).
2. K. Hornung, *Adiabatic and Isothermal Compressibility in the Liquid State*, Chapter 6.4 in **Handbook of Thermodynamic and Transport Properties of Alkali Metals**, R. W. Ohse, Editor, International Union of Pure and Applied Chemistry, Blackwell Science Publications, Boston (1985).
3. P. I. Bystrov, D. N. Kagan, G. A. Krechetova, and E. E. Shpil'rain, *Liquid-Metal Coolants for Heat Pipes and Power Plants*, ed, V. A. Kirillin, Hemisphere Pub. Corp., New York (1990).
4. P. Browning and P. E. Potter, *An Assessment of the Experimentally Determined Vapour Pressures of the Liquid Alkali Metals*, Chapter 6.2 in **Handbook of Thermodynamic and Transport Properties of Alkali Metals**, R. W. Ohse, Editor, International Union of Pure and Applied Chemistry, Blackwell Scientific Publications, Boston (1985).
5. G. H. Golden and T. V. Tokar, *Thermophysical Properties of Sodium*, **ANL-7323**, Argonne National Laboratory Report (1967).
6. J. D. Cox, D. D. Wagman, and V. A. Medvedev, *CODATA Key Values for Thermodynamics*, Hemisphere Publishing Corp., New York (1989).
7. E. H. P. Cordfunke and R. J. M. Konings, **Thermochemical Data for Reactor Materials and Fission Products**, North Holland Elsevier Science Pub. Co. Inc., New York (1990).
8. L. Leibowitz, M. G. Chasanov, and R. Blomquist, **J. Appl. Phys.** **42**, 2135 (1971).
9. M. G. Chasanov, L. Leibowitz, D. F. Fischer, and R. Blomquist, **J. Appl. Phys.** **43**, 748 (1972).

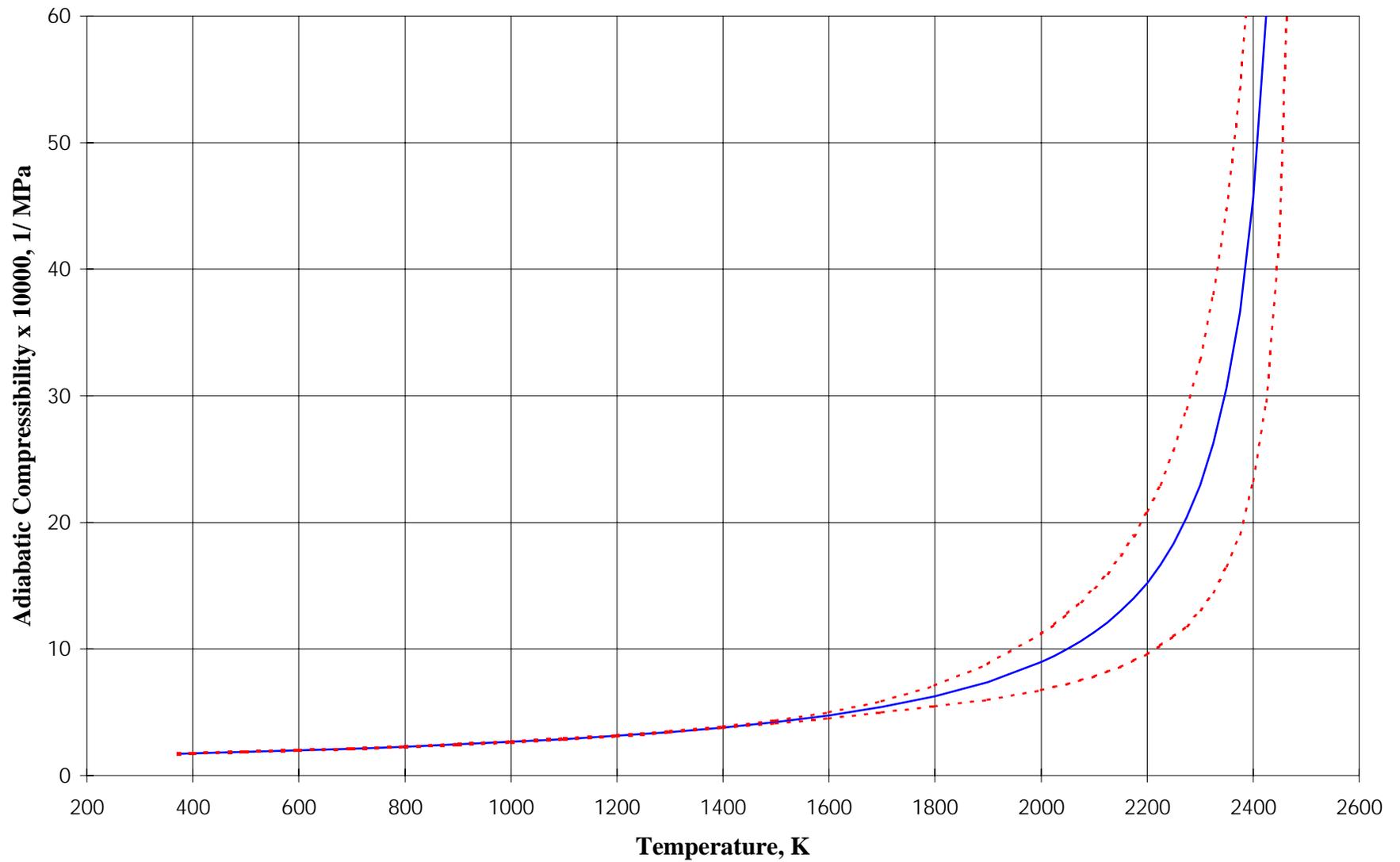


Fig. 1.4-1 Recommended Values for the Adiabatic Compressibility of Liquid Sodium

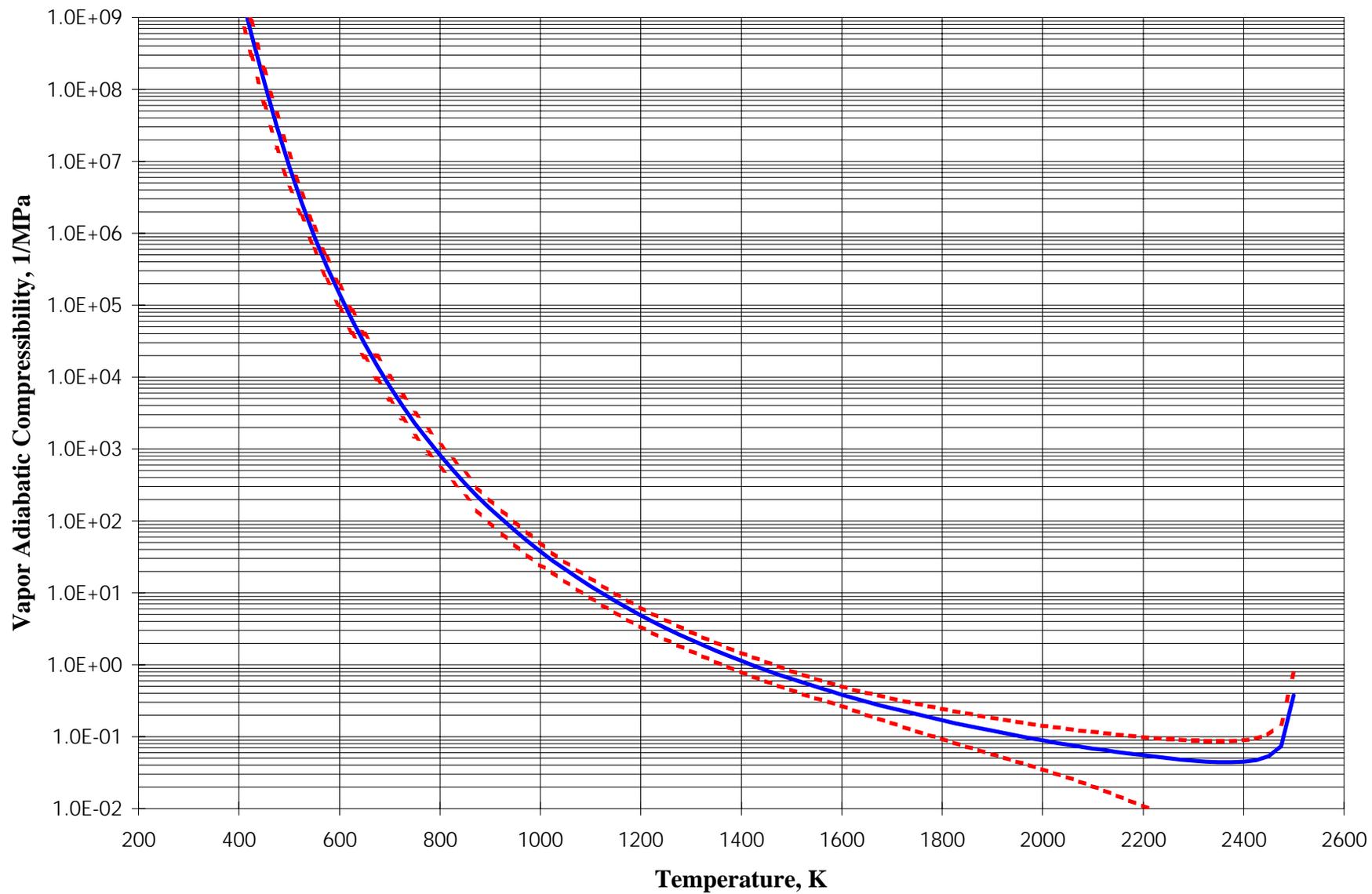


Fig.1.4-2 Recommended Values for the Adiabatic Compressibility of Sodium Vapor

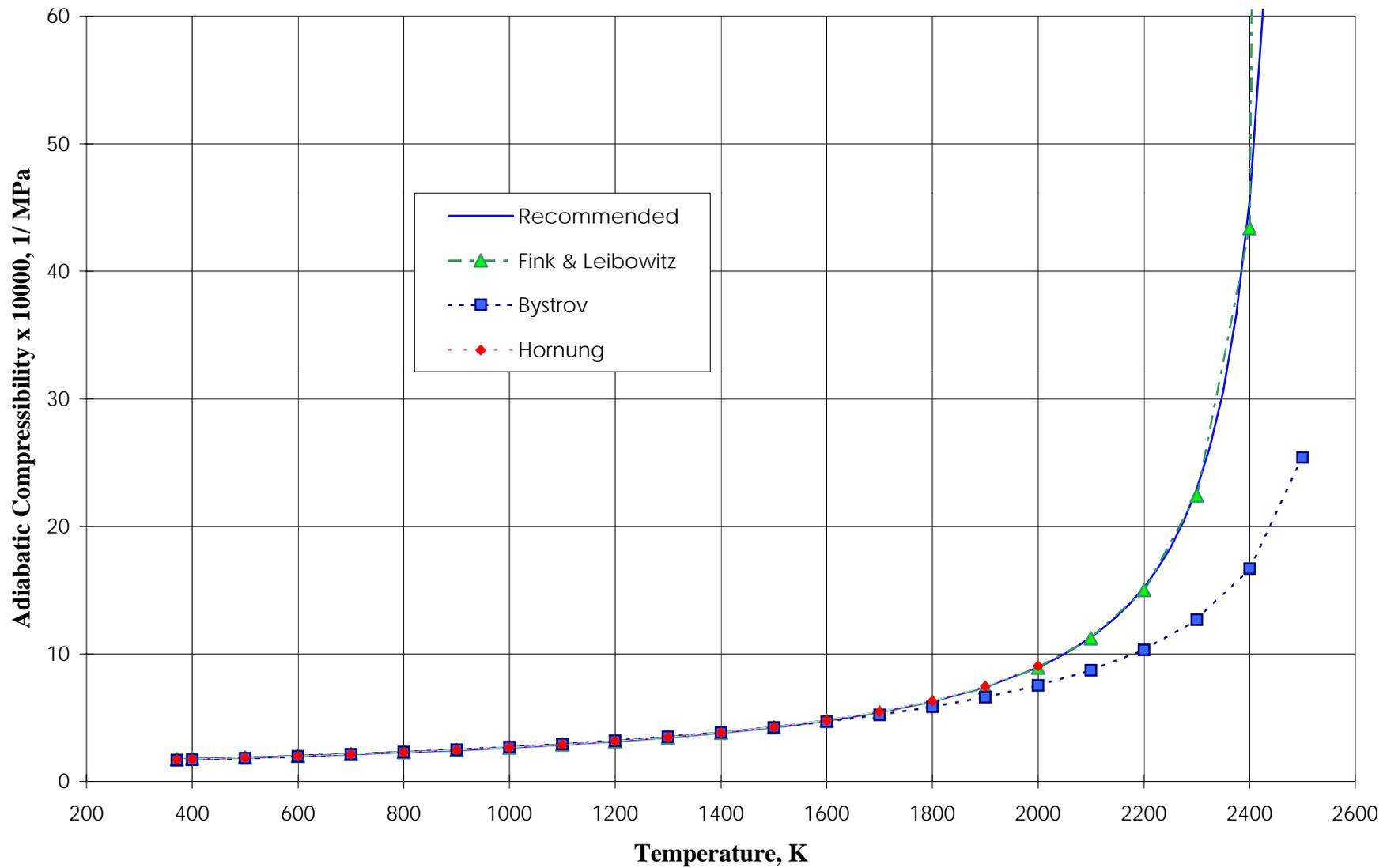


Fig. 1.4-3 Comparison of the Recommended Values for the Adiabatic Compressibility of Liquid Sodium with Values from Other Assessments

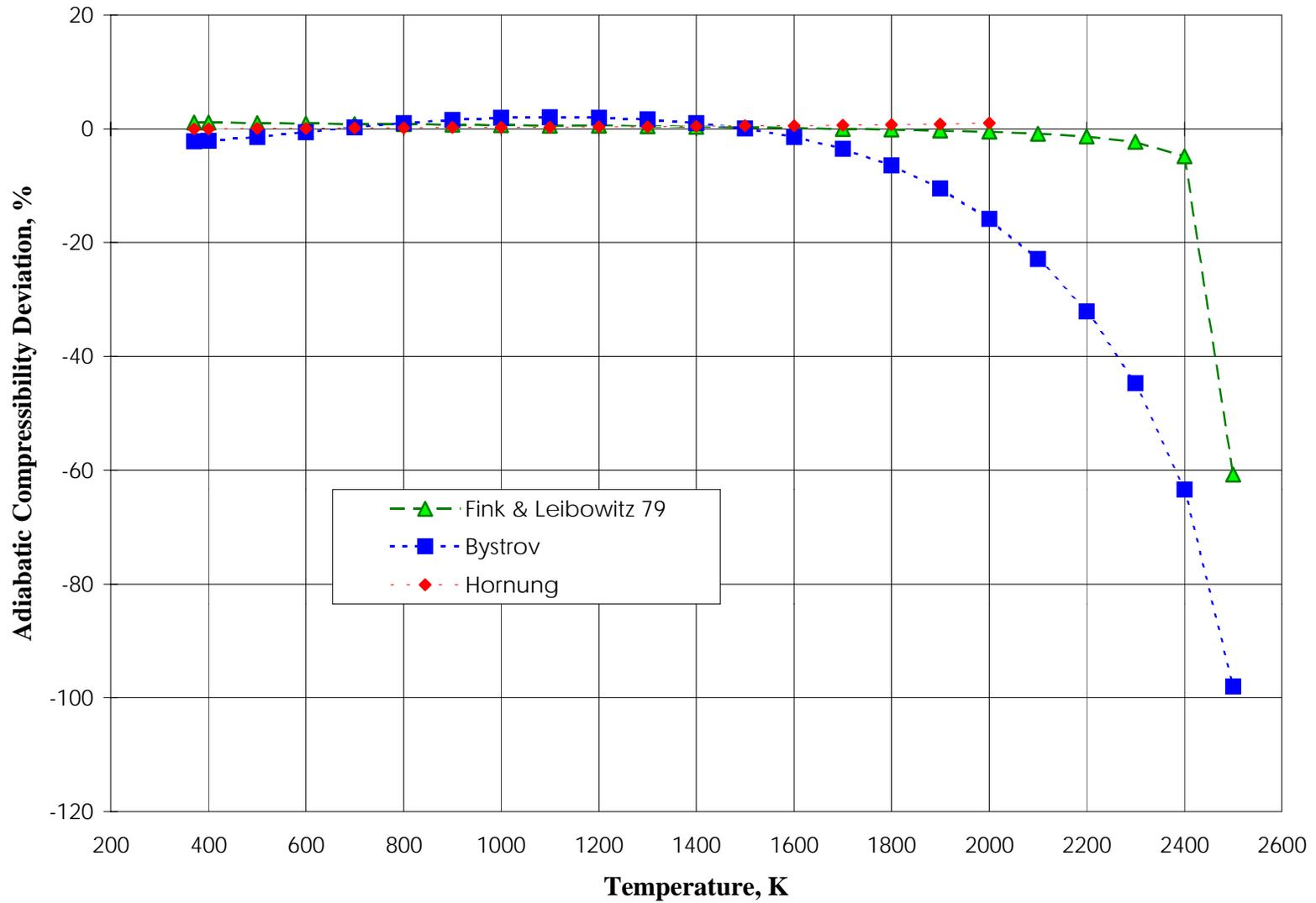


Fig. 1.4-4 Deviation of the Recommended Values for the Adiabatic Compressibility of Liquid Sodium from Values from Other Assessments

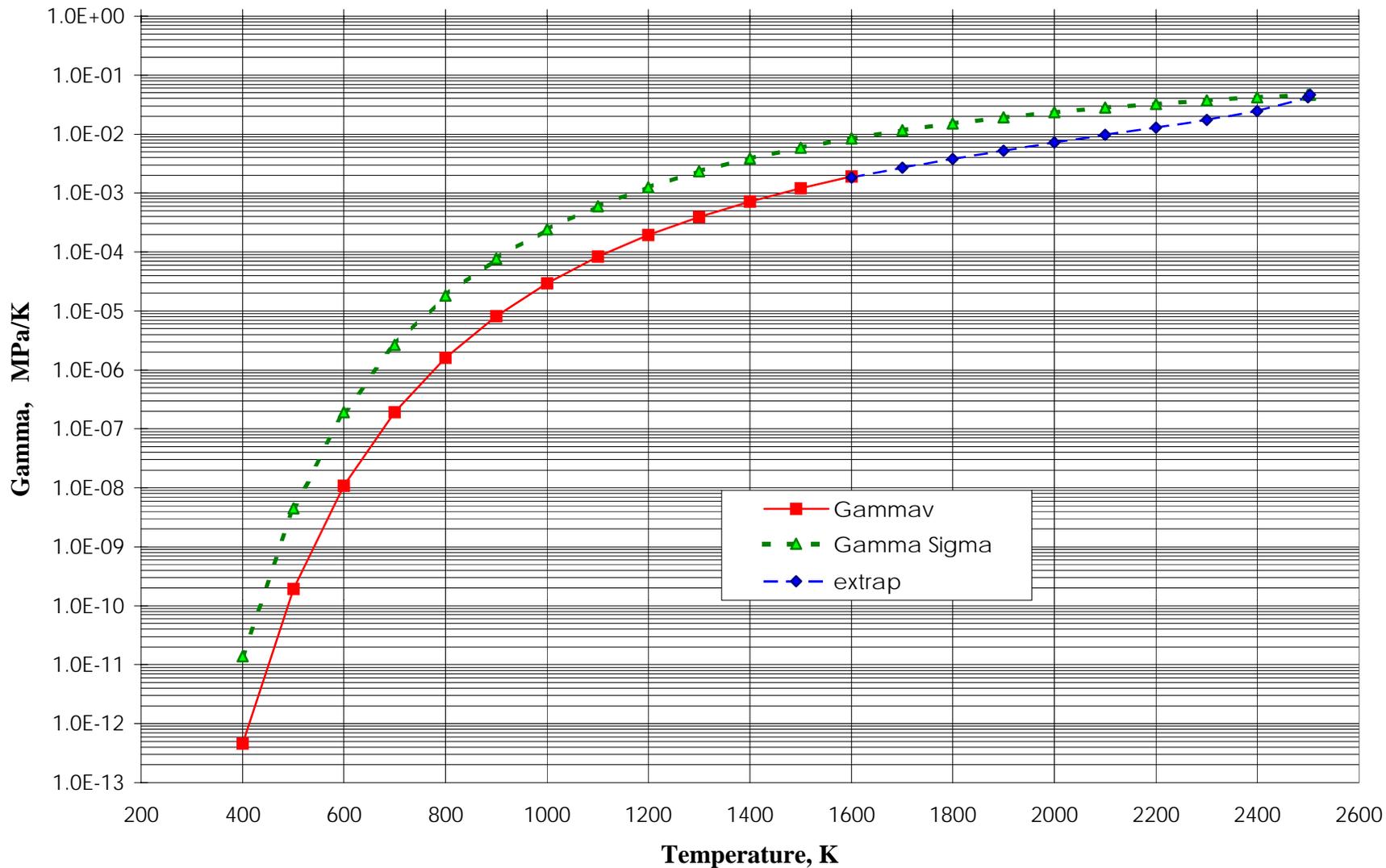


Fig. 1.4-5. The Thermal-Pressure Coefficient (γ_v) and the Temperature Derivative of the Vapor Pressure along the Saturation Curve (γ_σ)

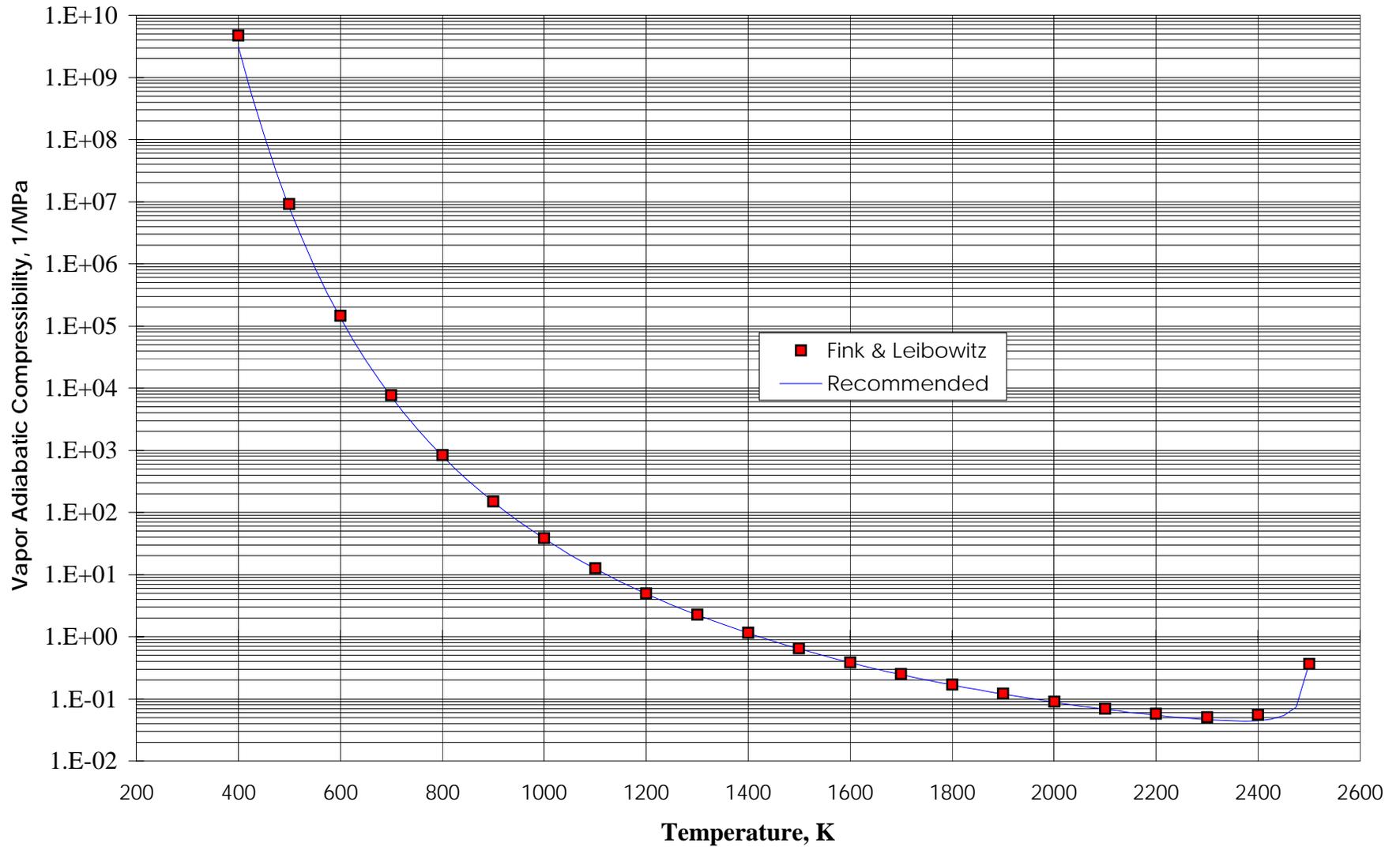


Fig. 1.4-6 Comparison of Recommended Values for the Adiabatic Compressibility of Sodium Vapor with Values from Fink and Leibowitz¹

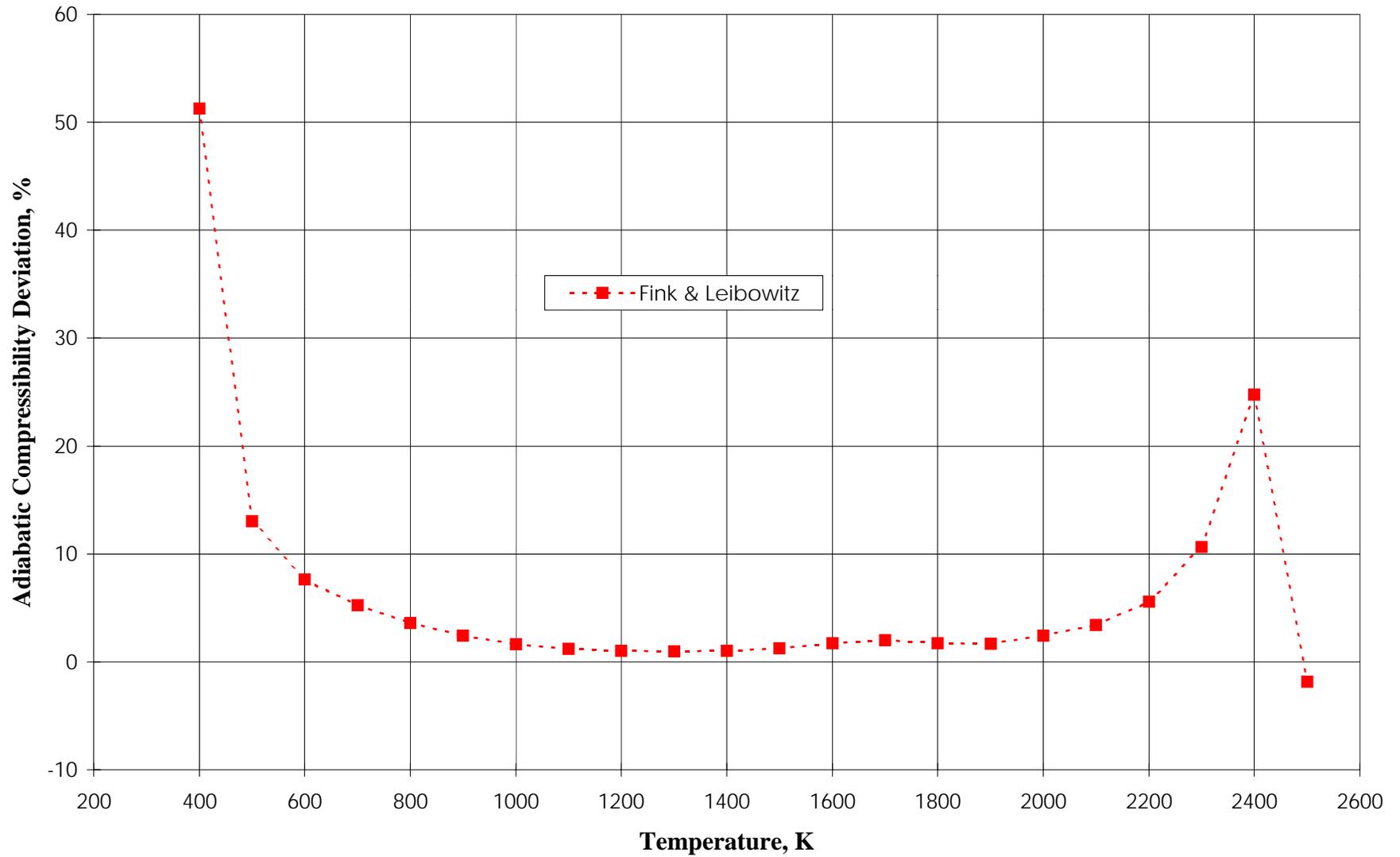


Fig. 1.4-7 Deviations of the Adiabatic Compressibility of sodium Vapor Calculated by Fink and Leibowitz¹ from Recommended Values

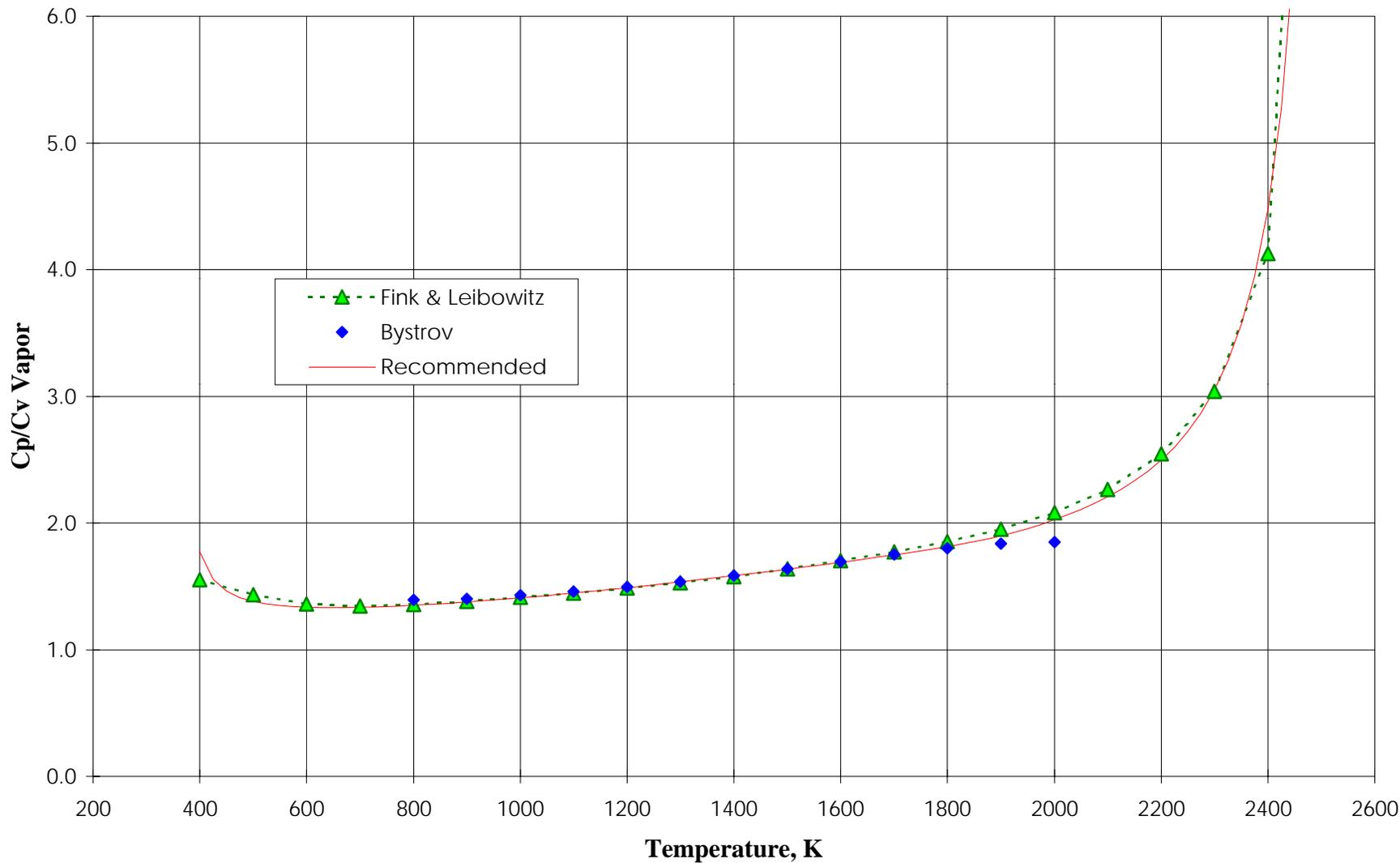


Fig. 1.4-8 Comparison of the Ratio of the Recommended Values for the Heat Capacity at Constant Pressure and Constant Volume for Sodium Vapor with Values of the Ratio from Other Assessments

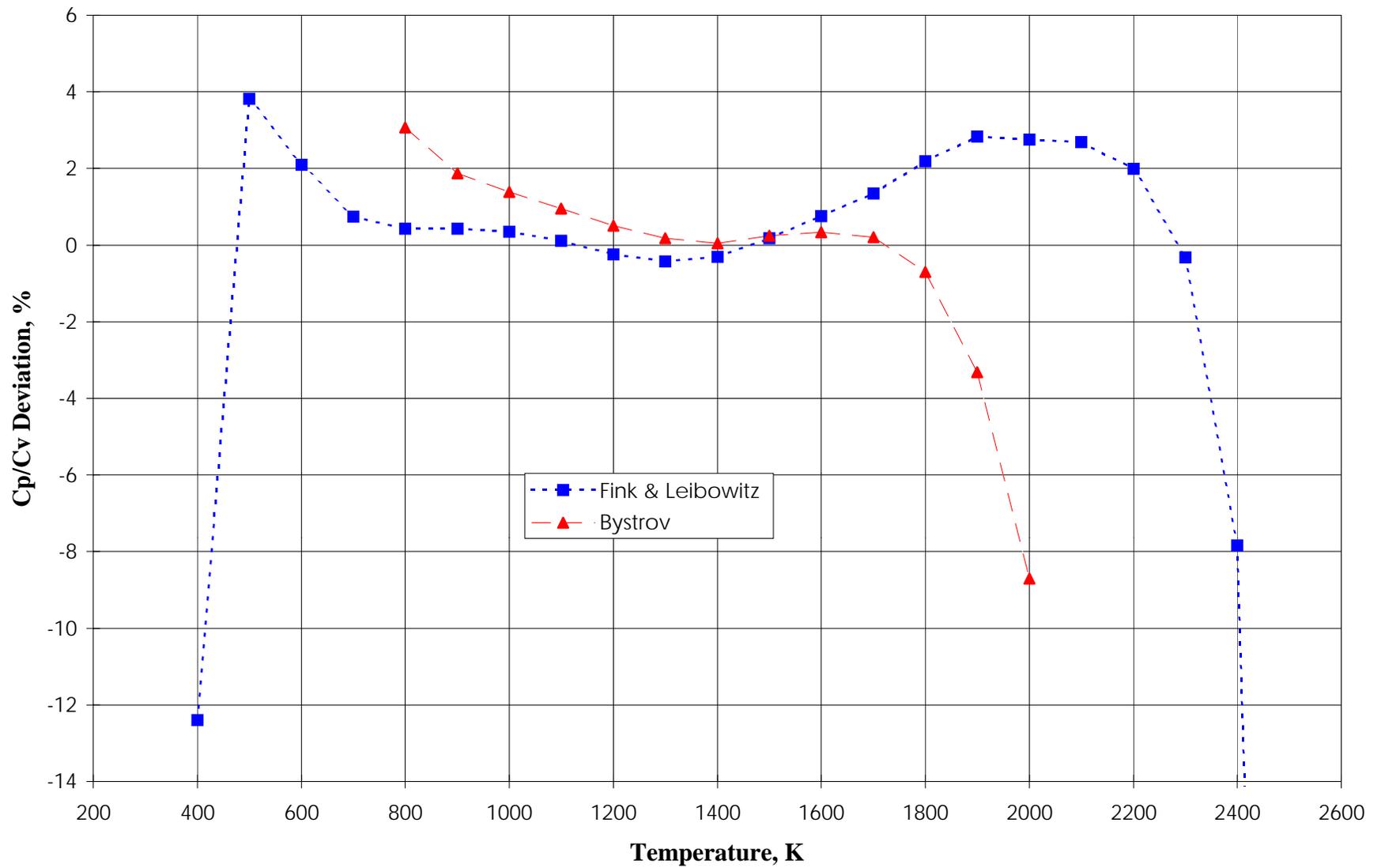


Fig. 1.4-9 Deviations of the Values of the Heat Capacity Ratios for Sodium Vapor Calculated by Fink and Leibowitz and by Bystrov et al. from Recommended Values

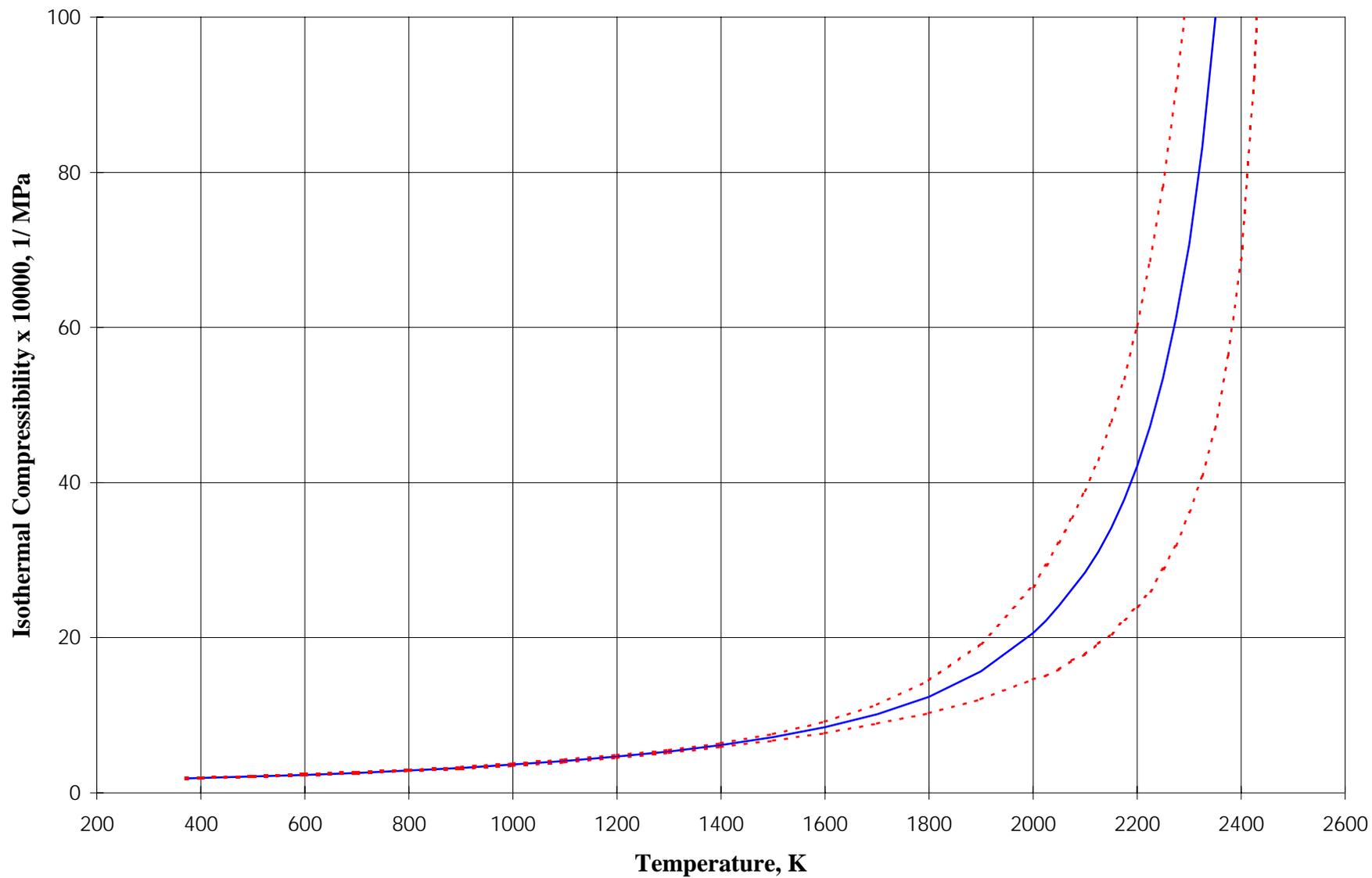


Fig. 1.4-10 Recommended Values for the Isothermal Compressibility of Liquid Sodium

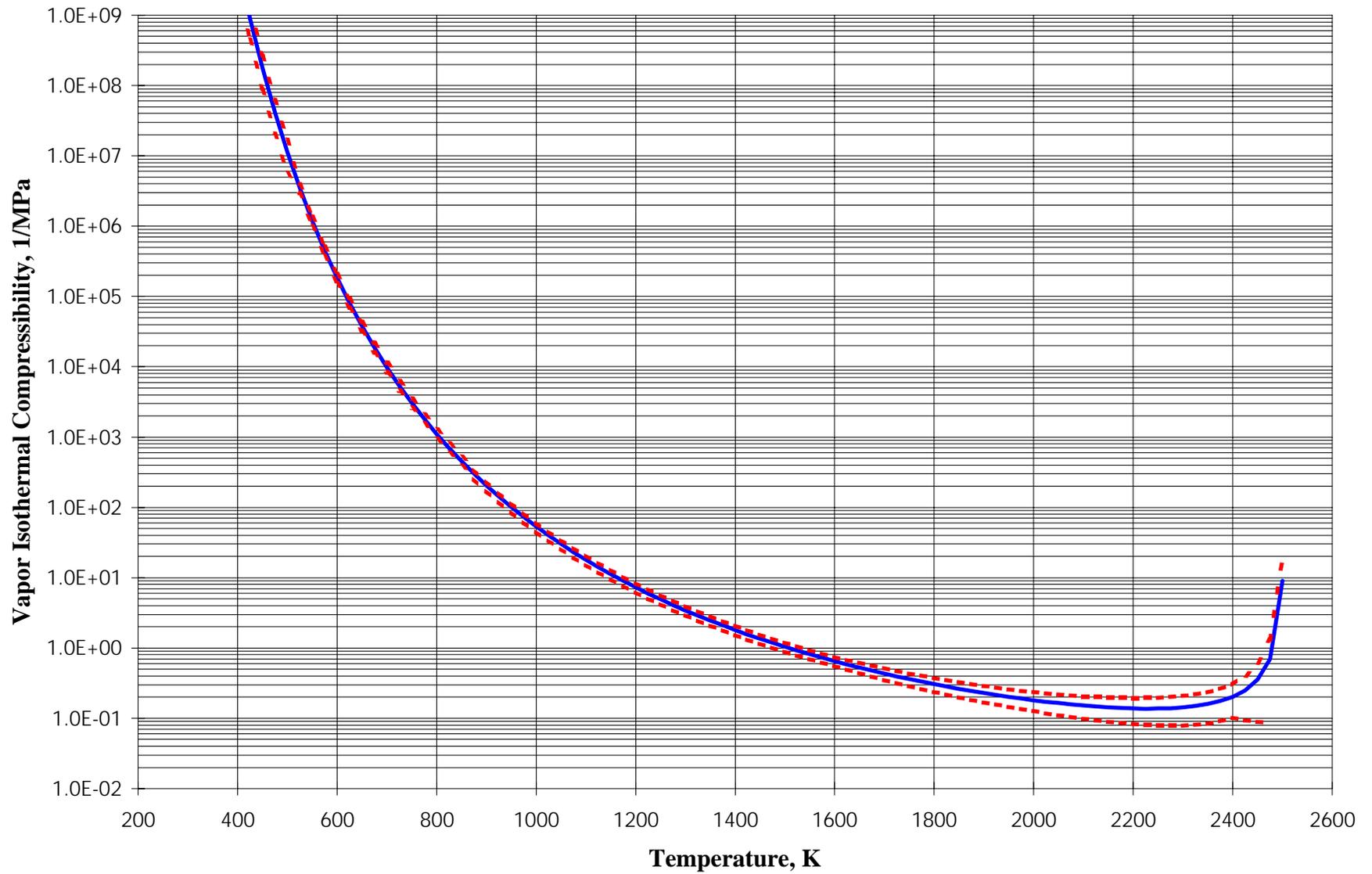


Fig. 1.4-11 Recommended Values for the Isothermal Compressibility of Sodium Vapor

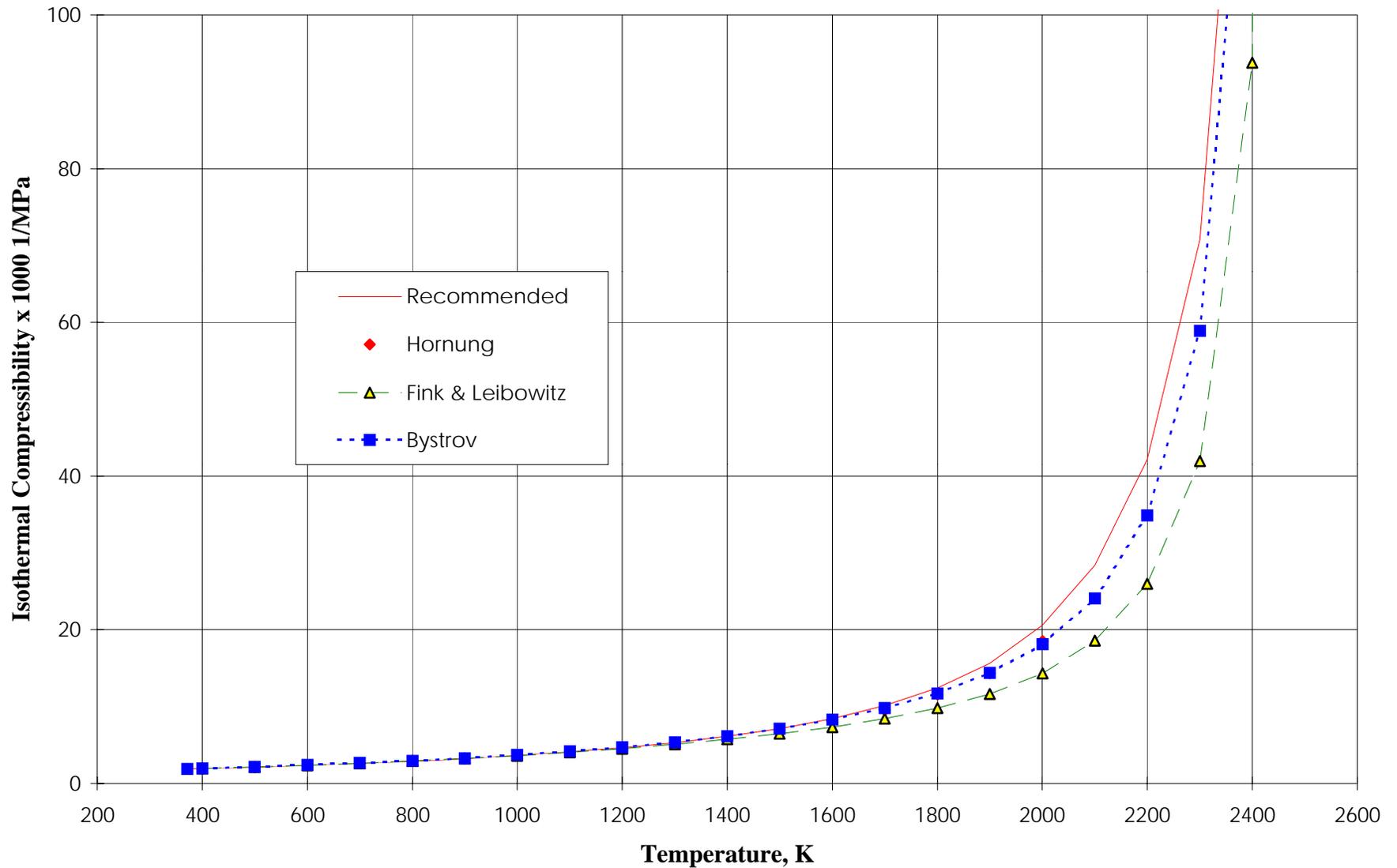


Fig. 1.4-12 Comparison of the Recommended Values for the Isothermal Compressibility of Liquid Sodium with Values from Other Assessments

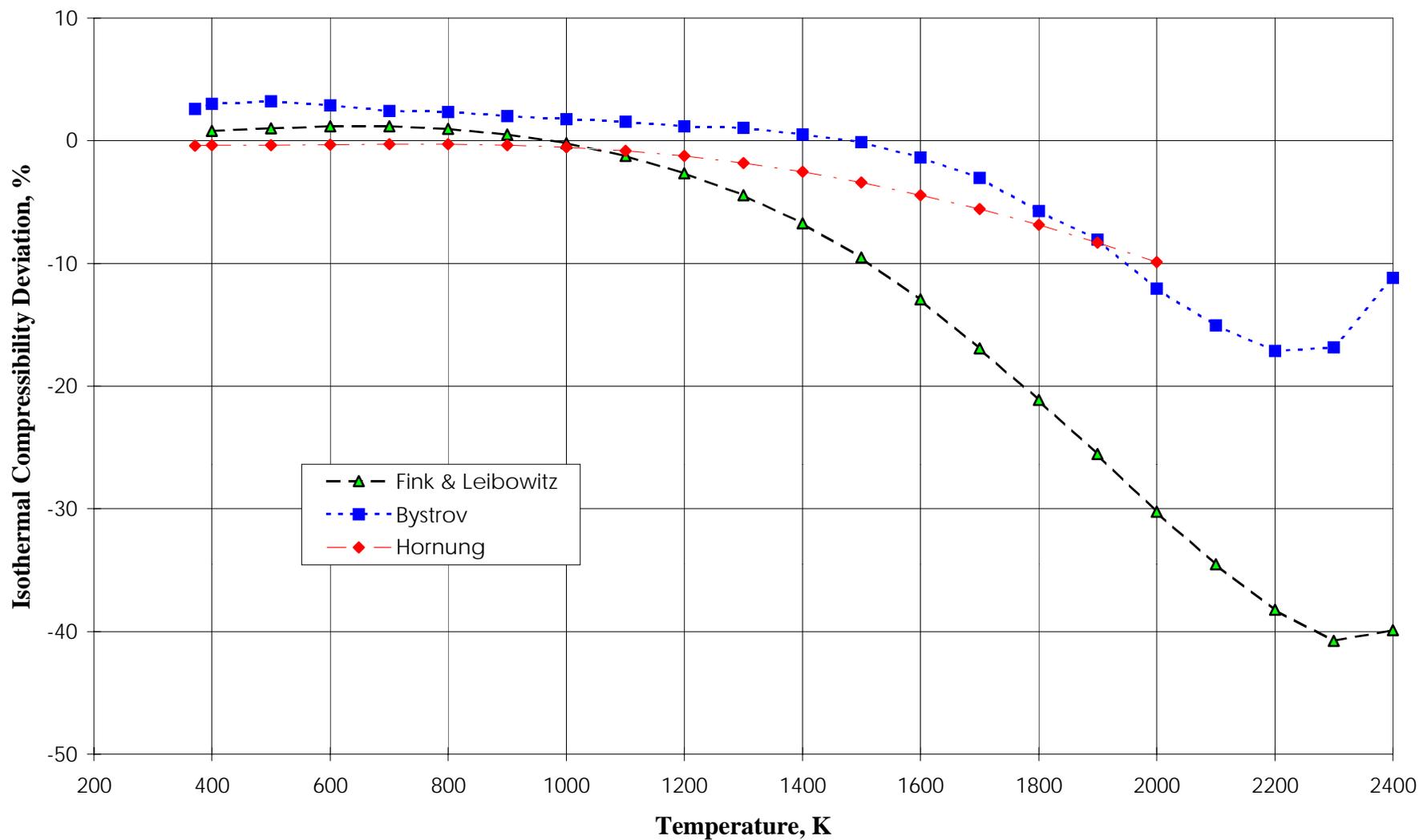


Fig. 1.4-13 Percent Deviations of the Recommended Values for the Isothermal Compressibility of Liquid Sodium from Values from Other Assessments

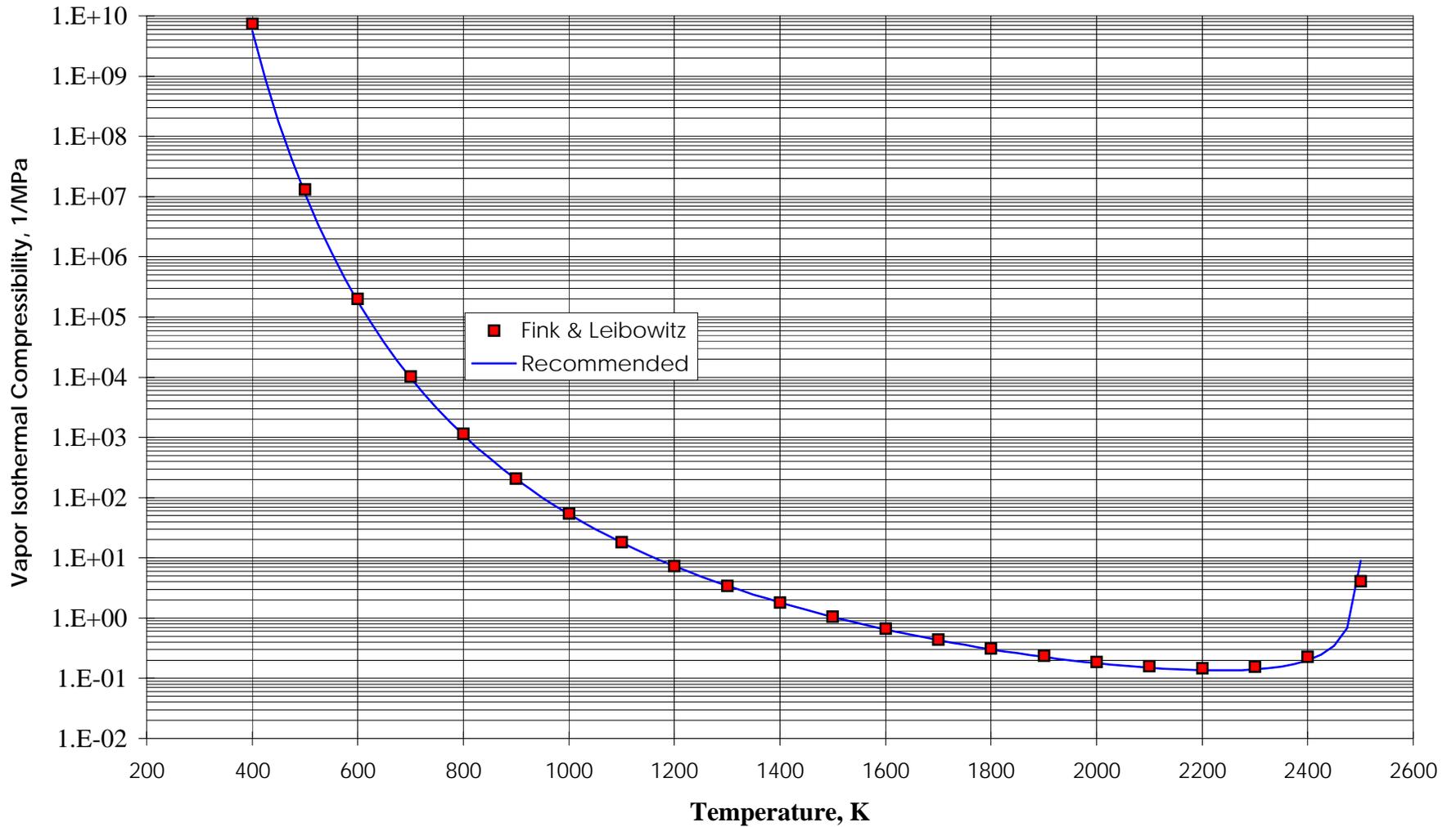


Fig. 1.4-14 Comparison of Recommended Values for the Isothermal Compressibility of Sodium Vapor with Values from Fink and Leibowitz¹

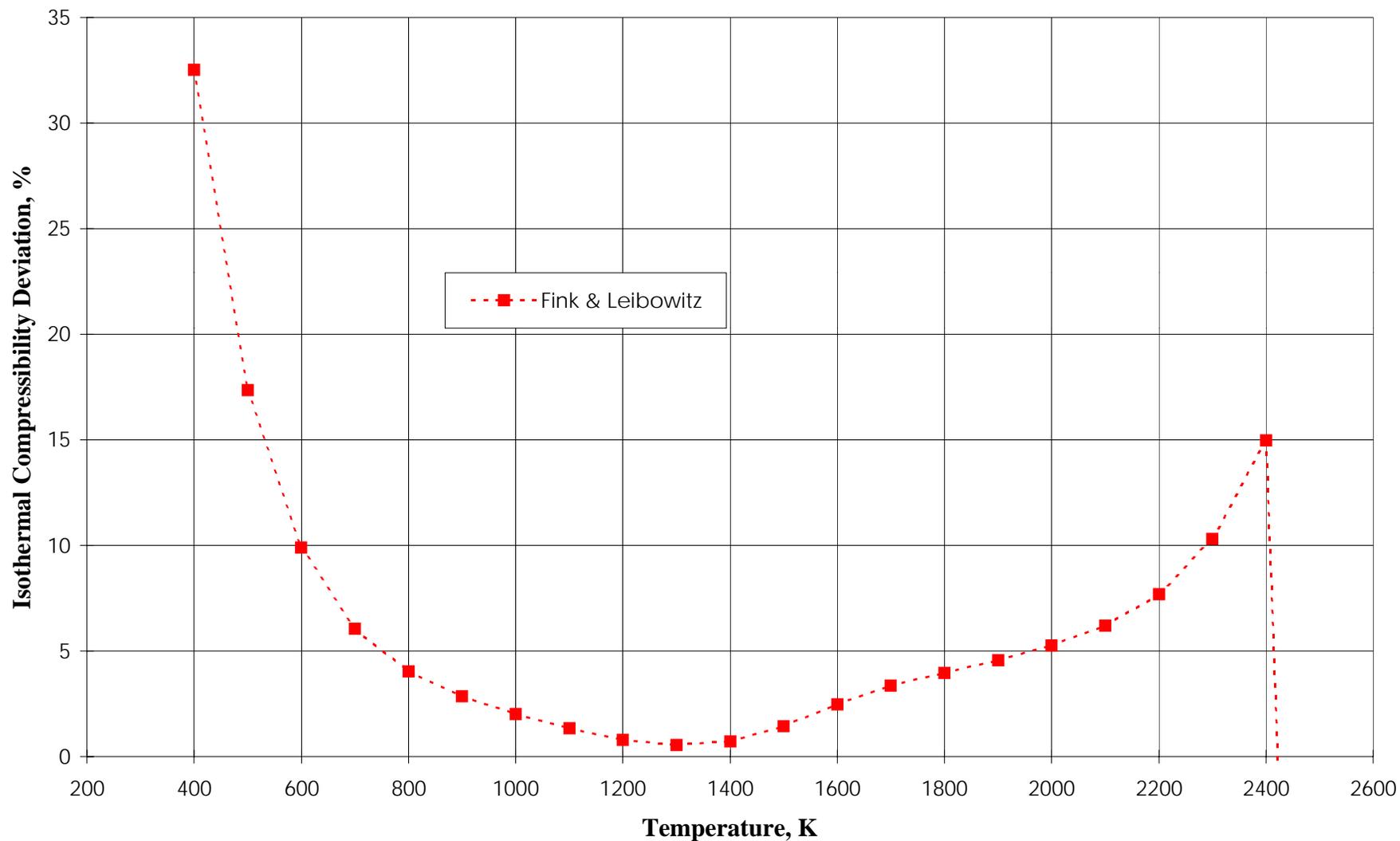


Fig. 1.4-15 Percent Deviations of the Recommended Values for the Isothermal Compressibility of Sodium Vapor from the Values from Fink and Leibowitz¹

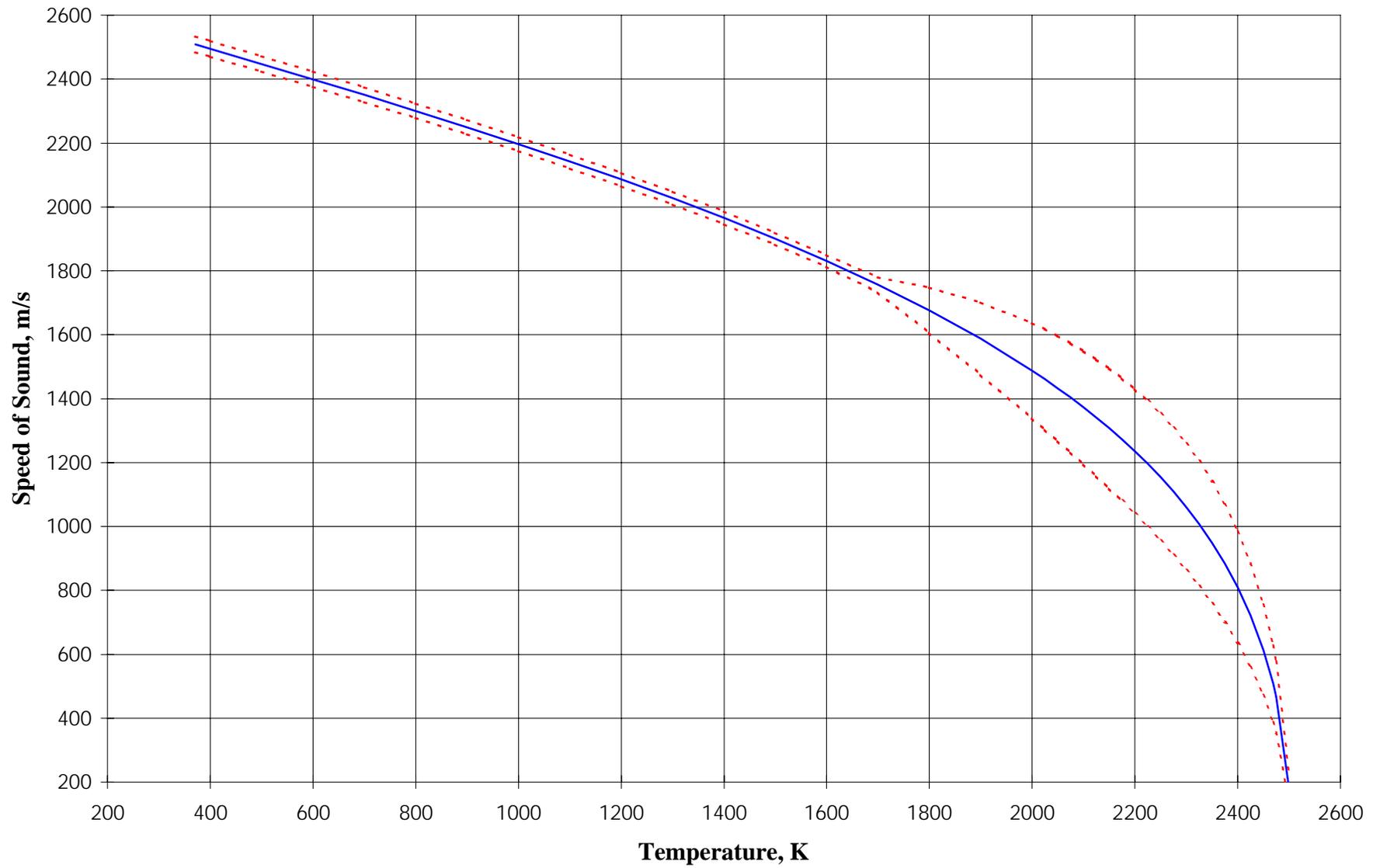


Fig. 1.4-16 Recommended Values for the Speed of Sound in Liquid Sodium

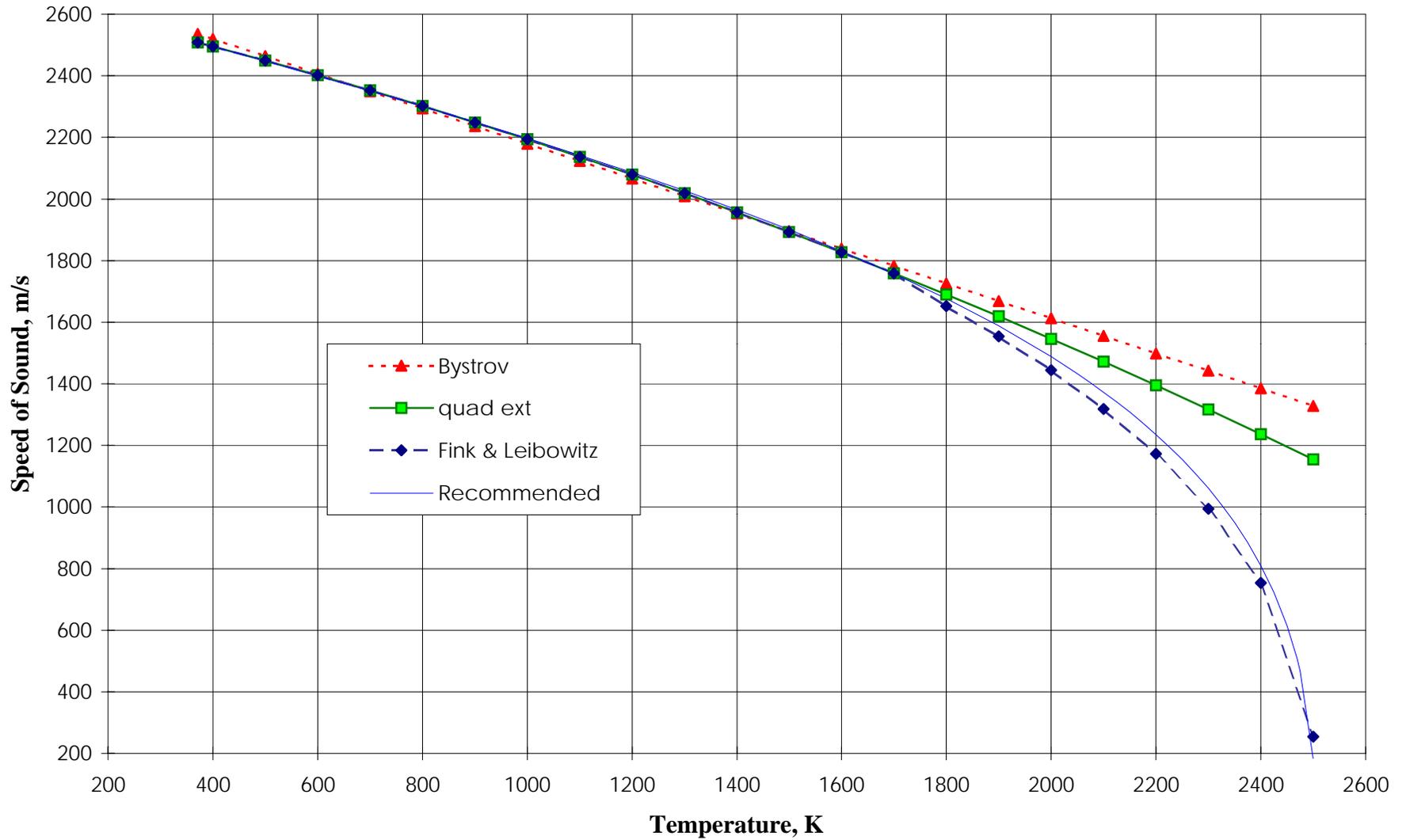


Fig. 1.4-17 Comparison of Recommended Values for the Speed of Sound in Liquid Sodium with Values from Other Assessments

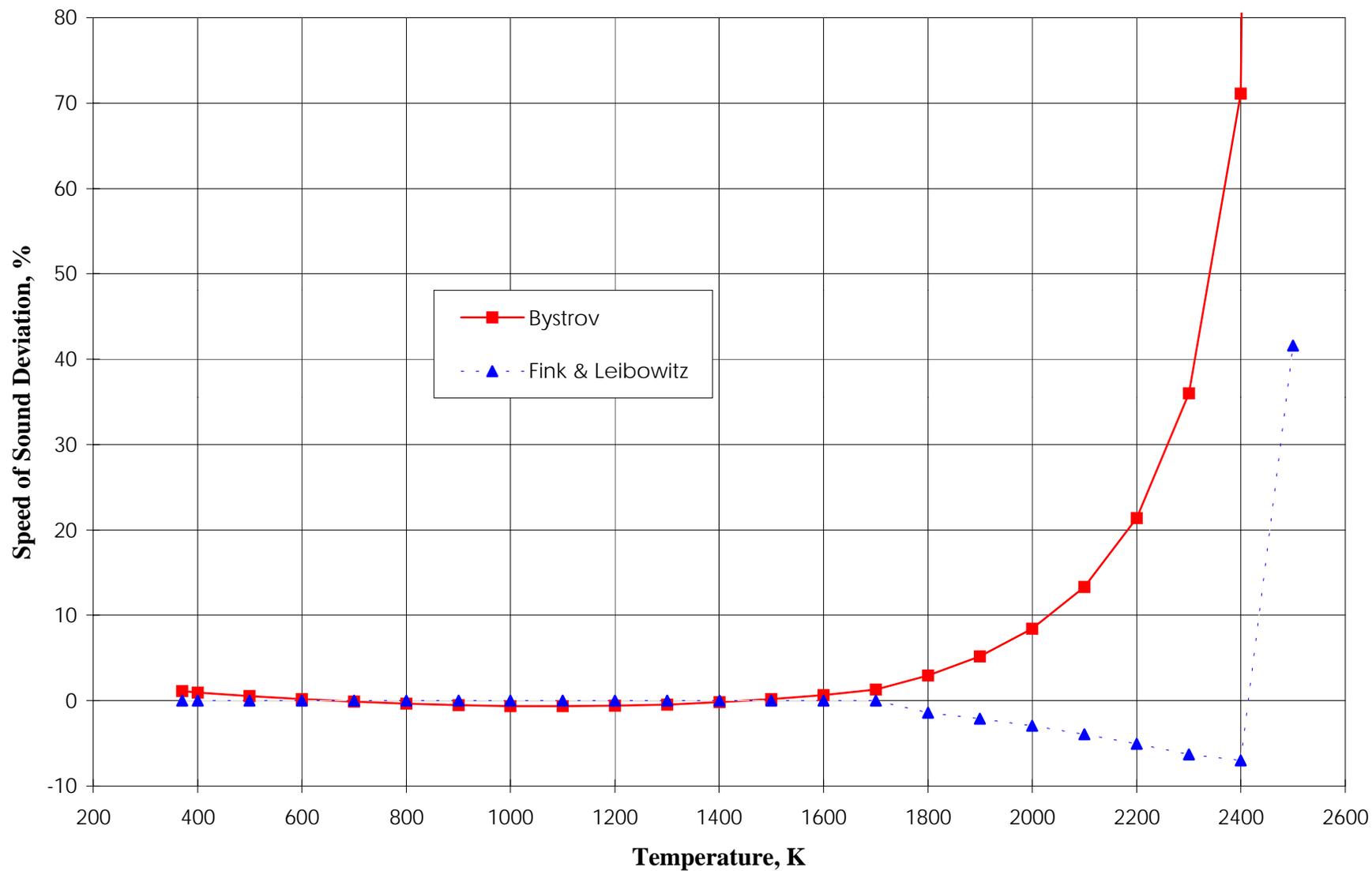


Fig. 1.4-18 Percent Deviations of the Recommended Values for the Speed of Sound in Liquid Sodium from Values from Other Assessments